

CERN-PH-TH/2010-065

UMN-TH-2840/10

FTPI-MINN-10/09

What if Supersymmetry Breaking Unifies beyond the GUT Scale?

John Ellis¹, Azar Mustafayev² and Keith A. Olive²

¹*TH Division, PH Department, CERN, CH-1211 Geneva 23, Switzerland*

²*William I. Fine Theoretical Physics Institute,
University of Minnesota, Minneapolis, MN 55455, USA*

Abstract

We study models in which soft supersymmetry-breaking parameters of the MSSM become universal at some unification scale, M_{in} , above the GUT scale, M_{GUT} . We assume that the scalar masses and gaugino masses have common values, m_0 and $m_{1/2}$ respectively, at M_{in} . We use the renormalization-group equations of the minimal supersymmetric SU(5) GUT to evaluate their evolutions down to M_{GUT} , studying their dependences on the unknown parameters of the SU(5) superpotential. After displaying some generic examples of the evolutions of the soft supersymmetry-breaking parameters, we discuss the effects on physical sparticle masses in some specific examples. We note, for example, that near-degeneracy between the lightest neutralino and the lighter stau is progressively disfavoured as M_{in} increases. This has the consequence, as we show in $(m_{1/2}, m_0)$ planes for several different values of $\tan \beta$, that the stau coannihilation region shrinks as M_{in} increases, and we delineate the regions of the $(M_{in}, \tan \beta)$ plane where it is absent altogether. Moreover, as M_{in} increases, the focus-point region recedes to larger values of m_0 for any fixed $\tan \beta$ and $m_{1/2}$. We conclude that the regions of the $(m_{1/2}, m_0)$ plane that are commonly favoured in phenomenological analyses tend to disappear at large M_{in} .

CERN-PH-TH/2010-065

March 2010

1 Introduction

The minimal supersymmetric extension of the Standard Model (MSSM) has over 100 free parameters, most of them associated with the breaking of supersymmetry [1–3]. Three classes of soft supersymmetry-breaking parameters are generally considered, the scalar masses m_0 , the gaugino masses $m_{1/2}$ and the trilinear scalar couplings A_0 , which are often assumed to be universal at some high input scale. Universality before renormalization of the m_0 parameters for different sfermions with the same electroweak quantum numbers is motivated by the upper limits on flavour-changing neutral interactions, and specific Grand Unified Theories (GUTs) suggest universality relations between squarks and sleptons [4]. Simple GUTs also favour universality before renormalization for the gaugino masses $m_{1/2}$, and universality is also a property of minimal supergravity (mSUGRA) models, which, however, also predict additional relations that we do not discuss here [5–7]. We refer to the scenario with universal m_0 , $m_{1/2}$ and A_0 as the constrained MSSM (CMSSM), and its parameter space has been explored extensively [8–14].

There is, however, one important question: at what renormalization scale M_{in} are m_0 and $m_{1/2}$ actually universal? The obvious possibility, and the one has been studied most frequently, is that universality applies at the same GUT scale, M_{GUT} , as coupling constant universality. In this case, the density of cold dark matter (assumed here to be composed mainly of the lightest neutralino, $\tilde{\chi}_1$, hereafter called χ) [15] is larger than the range favoured by WMAP [16] and other experiments in generic regions of the $(m_{1/2}, m_0)$ plane, and is compatible with WMAP only in narrow strips that are either close to the boundary where χ ceases to be the lightest sparticle – the stau [17] or stop [18] coannihilation strips – or close to the LEP2 chargino bound [19] – the bulk region [9, 15] – or where there is no electroweak symmetry breaking – the focus-point region [20] – or in a rapid-annihilation funnel (or A-funnel) [8].

However, it is not necessarily the case that $M_{in} = M_{GUT}$, since supersymmetry breaking might arise at some scale either below or even above M_{GUT} , and both possibilities have been studied in the literature. For example, as M_{in} is decreased below M_{GUT} , the differences between the renormalized sparticle masses diminish and the regions of the $(m_{1/2}, m_0)$ planes that yield the appropriate density of cold dark matter move away from the boundaries [21]. Eventually, for small M_{in} , the coannihilation and focus-point regions of the conventional GUT-scale CMSSM merge. Finally, for very small M_{in} they disappear entirely, and the relic χ density falls before the WMAP range everywhere in the $(m_{1/2}, m_0)$ plane, except for very large values of $m_{1/2}$ and m_0 .

What happens to the supersymmetric parameter space and sparticle phenomenology when $M_{in} > M_{GUT} \sim 2 \times 10^{16}$ GeV? Here, we consider values of M_{in} ranging up to the reduced Planck mass $\overline{M}_P \equiv M_P/\sqrt{2\pi} \sim 2.4 \times 10^{18}$ GeV. Generically, increasing M_{in} increases the renormalization of the sparticle masses which tends in turn to increase the splittings between the physical sparticle masses [22]. As we discuss in more detail below, this in turn has the effect of increasing the relic density in much of the $(m_{1/2}, m_0)$ plane. As a consequence, the coannihilation strip is squeezed to lower values of $m_{1/2}$ ¹, particularly for $\tan\beta \sim 10$, and

¹For a previous example of this phenomenon, see Fig. 5 of [23] or Fig. 3 of [24].

even disappears as M_{in} increases. At the same time, the focus-point strip often moves out to ever larger values of m_0 . There are also changes in the impacts of important constraints such as $g_\mu - 2$, $b \rightarrow s\gamma$ and m_h , which we also discuss below. The general conclusion is that the supersymmetric landscape would look rather different for $M_{in} > 10^{17}$ GeV from the CMSSM in which the universality scale $M_{in} = M_{GUT}$. The allowed region of parameter space that survives longest is the rapid-annihilation funnel at large $m_{1/2}$ and $\tan\beta$, which is compatible with the m_h and $b \rightarrow s\gamma$ constraints. In the CMSSM, the funnel region also requires large m_0 and would make a contribution to $g_\mu - 2$ that is too small to explain the experimental discrepancy with Standard Model calculations based on low-energy e^+e^- data. However, as we shall show, for large M_{in} , the funnel region extends to low m_0 (including $m_0 = 0$) and in some cases will be compatible with the $g_\mu - 2$ measurements.

We underline that there are some potential ambiguities in these conclusions. We use for our analysis above M_{GUT} the particle content and the renormalization-group equations (RGEs) of the minimal SU(5) GUT [22, 25], primarily for simplicity and so as to minimize the number of additional parameters to be explored: for a recent review of this sample model and its compatibility with experiment, see [26]. Even in this simplest GUT, there are two couplings in the SU(5) superpotential that make potentially significant contributions to the RGE running but are poorly constrained. We explore their impacts on our results, and find that one of the couplings could have noticeable effects in the focus-point region, if it is large. Secondly, we are aware that the minimal SU(5) model is surely inadequate; for example, it does not include neutrino masses. The effects of a minimal seesaw sector on the GUT RGEs [27] and on the relic density [23, 24, 28] has been considered elsewhere: they also may be small if the neutrino Dirac Yukawa couplings are not large. However, the minimal SU(5) GUT also has issues with proton decay via dimension-five operators, which may be alleviated if the GUT triplet Higgs particles are relatively heavy, as would happen if the associated SU(5) superpotential coupling were large [29]. We therefore consider this option as the default in our analysis. One could in principle consider non-minimal GUT models in which dimension-five proton decay is suppressed by some other mechanism, but the exploration of such models would take us too far from our objective here.

The layout of this paper is as follows. In Section 2, we recall the superpotential of the minimal SU(5) GUT and the corresponding RGEs for the soft supersymmetry-breaking parameters. Here, we give some examples of the RGE running, assuming universality at some high scale $M_{in} > M_{GUT}$. We also give examples of the dependences of physical sparticle masses on M_{in} , illustrating features that are important for understanding qualitatively the dependences of features in the $(m_{1/2}, m_0)$ plane. Several of these are displayed in Section 3, for representative values of M_{in} and default values of the unknown SU(5) superpotential parameters. As already mentioned, two of the striking features in these planes are the disappearance of the coannihilation strip and the movement of the focus-point strip to larger m_0 as M_{in} increases. We discuss in Section 4 the sensitivities of these features to the choices of the SU(5) superpotential parameters, showing that the disappearance of the coannihilation strip is relatively model-independent, whereas the movement of the focus-point strip is more model-dependent. Finally, in Section 5 we summarize our results and draw some conclusions for the generalization of the standard CMSSM with $M_{in} = M_{GUT}$ to different values of M_{in} .

2 The Minimal SU(5) GUT Superpotential and RGEs

In the SU(5) GUT, the \hat{D}_i^c and \hat{L}_i superfields of the MSSM reside in the $\bar{\mathbf{5}}$ representation, $\hat{\phi}_i$, while the \hat{Q}_i , \hat{U}_i^c and \hat{E}_i^c superfields are in the $\mathbf{10}$ representation, $\hat{\psi}_i$. In the minimal scenario, one introduces a single SU(5) adjoint Higgs multiplet $\hat{\Sigma}(\mathbf{24})$, and the two Higgs doublets of the MSSM, \hat{H}_d and \hat{H}_u are extended to five-dimensional SU(5) representations $\hat{\mathcal{H}}_1(\bar{\mathbf{5}})$ and $\hat{\mathcal{H}}_2(\mathbf{5})$ respectively. The minimal renormalizable superpotential for this model is

$$W_5 = \mu_\Sigma \text{Tr} \hat{\Sigma}^2 + \frac{1}{6} \lambda' \text{Tr} \hat{\Sigma}^3 + \mu_H \hat{\mathcal{H}}_{1\alpha} \hat{\mathcal{H}}_2^\alpha + \lambda \hat{\mathcal{H}}_{1\alpha} \hat{\Sigma}_\beta^\alpha \hat{\mathcal{H}}_2^\beta \\ + (\mathbf{h}_{10})_{ij} \epsilon_{\alpha\beta\gamma\delta\zeta} \hat{\psi}_i^{\alpha\beta} \hat{\psi}_j^{\gamma\delta} \hat{\mathcal{H}}_2^\zeta + (\mathbf{h}_{\bar{5}})_{ij} \hat{\psi}_i^{\alpha\beta} \hat{\phi}_{j\alpha} \hat{\mathcal{H}}_{1\beta} \quad (1)$$

where Greek letters denote SU(5) indices, $i, j = 1..3$ are generation indices and ϵ is the totally antisymmetric tensor with $\epsilon_{12345} = 1$. This simple model predicts (approximately) correctly the observed ratio of the τ and b quark masses, but the corresponding predictions for the lighter charged-lepton and charge -1/3 quark masses are at best qualitatively successful. It is possible to add to (1) terms that are non-renormalizable quartic and of higher order in the Higgs fields that could rectify these less successful predictions: such terms would not contribute to the RGEs and low-energy observables that we study. In this paper, we will work in the third-generation-dominance scheme where Yukawas of first two generations are neglected, *i.e.*, we assume $\mathbf{h}_{\bar{5},10} \sim (\mathbf{h}_{\bar{5},10})_{33} \equiv h_{\bar{5},10}$.

We work in a vacuum that breaks $\text{SU}(5) \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$, in which $\langle \hat{\Sigma} \rangle = v_{24} \text{Diag}(2, 2, 2, -3, -3)$ and the GUT gauge bosons acquire masses $M_{X,Y} = 5g_{GUT}v_{24}$. The fine-tuning condition $\mu_\Sigma - 3\lambda v_{24} = \mathcal{O}(M_Z)$ must be imposed in order to obtain the gauge hierarchy, in which case the triplet Higgs states have masses $M_{H_3} = \lambda v_{24}/g_{GUT}$. The amplitude for proton decay via a dimension-five operator $\propto 1/M_{H_3}$, and so is relatively suppressed for large λ . However, the amplitude also depends on other model parameters, so it is difficult to quantify this argument, which would in any case be avoided in suitable non-minimal SU(5) models. In this paper we compare results for the values $\lambda = 1$ and 0.1, treating the former as our default value.

The RGEs for the Yukawa couplings in the superpotential (1) that are applicable between M_{in} and M_{GUT} are:

$$\frac{dh_{\bar{5}}}{dt} = \frac{h_{\bar{5}}}{16\pi^2} \left[5h_{\bar{5}}^2 + 48h_{10}^2 + \frac{24}{5}\lambda^2 - \frac{84}{5}g_5^2 \right], \quad (2)$$

$$\frac{dh_{10}}{dt} = \frac{h_{10}}{16\pi^2} \left[144h_{10}^2 + 2h_{\bar{5}}^2 + \frac{24}{5}\lambda^2 - \frac{96}{5}g_5^2 \right], \quad (3)$$

$$\frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2} \left[48h_{10}^2 + 2h_{\bar{5}}^2 + \frac{53}{5}\lambda^2 + \frac{21}{20}\lambda'^2 - \frac{98}{5}g_5^2 \right], \quad (4)$$

$$\frac{d\lambda'}{dt} = \frac{\lambda'}{16\pi^2} \left[3\lambda^2 + \frac{63}{20}\lambda'^2 - 30g_5^2 \right], \quad (5)$$

where g_5 is the SU(5) gauge coupling above the GUT scale. We note that the Yukawa coupling λ contributes directly to the RGEs for $h_{\bar{5}}$ and h_{10} while λ' contributes indirectly

through its effect on the running of λ . In most analyses of the CMSSM, $h_{\mathbf{\bar{5}}}$ and $h_{\mathbf{10}}$ are chosen at the GUT scale so as to reproduce the measured t and b masses. Equations (4) and (5) tell us that the input values required at M_{in} depend on λ . Other quantities renormalized by $h_{\mathbf{\bar{5}}}$ and $h_{\mathbf{10}}$, such as the third-generation tenplet mass $m_{\mathbf{10}}$ and fiveplet mass $m_{\mathbf{\bar{5}}}$, will thereby acquire some indirect dependence on λ .

The RGEs for the most relevant soft supersymmetry-breaking squared scalar masses between M_{in} and M_{GUT} are:

$$\frac{dm_{\mathbf{\bar{5}}}^2}{dt} = \frac{1}{8\pi^2} \left[2h_{\mathbf{\bar{5}}}^2 \{m_{\mathcal{H}_1}^2 + m_{\mathbf{10}}^2 + m_{\mathbf{\bar{5}}}^2 + A_{\mathbf{\bar{5}}}^2\} - \frac{48}{5} g_5^2 M_5^2 \right], \quad (6)$$

$$\begin{aligned} \frac{dm_{\mathbf{10}}^2}{dt} = \frac{1}{8\pi^2} & \left[48h_{\mathbf{10}}^2 \{m_{\mathcal{H}_2}^2 + 2m_{\mathbf{10}}^2 + A_{\mathbf{10}}^2\} \right. \\ & \left. + h_{\mathbf{\bar{5}}}^2 \{m_{\mathcal{H}_1}^2 + m_{\mathbf{10}}^2 + m_{\mathbf{\bar{5}}}^2 + A_{\mathbf{\bar{5}}}^2\} - \frac{72}{5} g_5^2 M_5^2 \right], \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{dm_{\mathcal{H}_1}^2}{dt} = \frac{1}{8\pi^2} & \left[2h_{\mathbf{\bar{5}}}^2 \{m_{\mathcal{H}_1}^2 + m_{\mathbf{10}}^2 + m_{\mathbf{\bar{5}}}^2 + A_{\mathbf{\bar{5}}}^2\} \right. \\ & \left. + \frac{24}{5} \lambda^2 \{m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_{\Sigma}^2 + A_{\lambda}^2\} - \frac{48}{5} g_5^2 M_5^2 \right], \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{dm_{\mathcal{H}_2}^2}{dt} = \frac{1}{8\pi^2} & \left[48h_{\mathbf{10}}^2 \{m_{\mathcal{H}_2}^2 + 2m_{\mathbf{10}}^2 + A_{\mathbf{10}}^2\} \right. \\ & \left. + \frac{24}{5} \lambda^2 \{m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_{\Sigma}^2 + A_{\lambda}^2\} - \frac{48}{5} g_5^2 M_5^2 \right], \end{aligned} \quad (9)$$

$$\frac{dm_{\Sigma}^2}{dt} = \frac{1}{8\pi^2} \left[\frac{21}{20} \lambda'^2 \{3m_{\Sigma}^2 + A_{\lambda'}^2\} + \lambda^2 \{m_{\mathcal{H}_1}^2 + m_{\mathcal{H}_2}^2 + m_{\Sigma}^2 + A_{\lambda}^2\} - 20g_5^2 M_5^2 \right]. \quad (10)$$

The RGEs for the first two generations' sfermion masses $m_{\mathbf{10},1}$ and $m_{\mathbf{\bar{5}},1}$ contain only gauge-gaugino parts, which are identical to their third-generation counterparts (6, 7). We note that all the scalar masses-squared are strongly renormalized by g_5 , some of whose implications we shall discuss later.

We see that the extra GUT superpotential couplings λ, λ' do not affect directly the evolutions of the soft supersymmetry-breaking masses of the matter multiplets. However, the value of λ may have important effects on the evolutions of the soft supersymmetry-breaking masses of the fiveplet Higgs multiplets. On the other hand, the value of λ' impacts directly only the adjoint Higgs multiplet, which then contributes in turn to the evolutions of the masses of the fiveplet Higgs multiplets. This leads us to expect that our results should be relatively insensitive to λ' . However, we do expect them to be sensitive to λ , particularly as one approaches the focus-point region at large m_0 , since it lies close to the boundary of consistent electroweak symmetry breaking (EWSB). We do not discuss here the RGEs for the trilinear soft supersymmetry-breaking parameters A_i : the patterns of their dependences on λ and λ' are similar to the RGEs for the scalar masses-squared. The complete set of RGEs

for minimal SU(5) can be found in Ref. [22,30] with appropriate changes of notation^{2,3}.

The model is specified by the following set of parameters

$$m_0, m_{1/2}, A_0, M_{in}, \lambda, \lambda', \tan \beta, \text{sign}(\mu) \quad (11)$$

where the trilinear superpotential Higgs couplings, λ, λ' , are specified at $Q = M_{GUT}$. Assuming universality at a unification scale M_{in} , we impose

$$\begin{aligned} m_{\bar{\mathbf{5}},1} = m_{\mathbf{10},1} = m_{\bar{\mathbf{5}}} = m_{\mathbf{10}} = m_{\mathcal{H}_1} = m_{\mathcal{H}_2} = m_{\Sigma} &\equiv m_0, \\ A_{\bar{\mathbf{5}}} = A_{\mathbf{10}} = A_{\lambda} = A_{\lambda'} &\equiv A_0, \\ M_5 &\equiv m_{1/2}. \end{aligned} \quad (12)$$

and evolve all parameters to M_{GUT} using the SU(5) RGEs mentioned earlier. Clearly, the CMSSM is realized by setting $M_{in} = M_{GUT}$, and in this case, the couplings λ and λ' have no effect on the low-energy spectrum. Bilinear superpotential parameters μ_H, μ_{Σ} and corresponding soft supersymmetry breaking terms decouple from the rest of RGEs and therefore are omitted. The transition to the MSSM is done at M_{GUT} via the following matching conditions:

$$\begin{aligned} g_1 = g_2 = g_3 = g_5, \quad h_t = 4h_{\mathbf{10}}, \\ M_1 = M_2 = M_3 = M_5, \\ m_{D_1}^2 = m_{L_1}^2 = m_{\bar{\mathbf{5}},1}^2, \quad m_{Q_1}^2 = m_{U_1}^2 = m_{E_1}^2 = m_{\mathbf{10},1}^2, \\ m_{D_3}^2 = m_{L_3}^2 = m_{\bar{\mathbf{5}}}^2, \quad m_{Q_3}^2 = m_{U_3}^2 = m_{E_3}^2 = m_{\mathbf{10}}^2, \\ m_{H_d}^2 = m_{\mathcal{H}_1}^2, \quad m_{H_u}^2 = m_{\mathcal{H}_2}^2. \end{aligned} \quad (13)$$

Note that we do not impose $b - \tau$ Yukawa unification at M_{GUT} ; although exact unification is possible in the MSSM, it is not guaranteed over the entire parameter space, and in a GUT non-renormalizable operators of the type needed to modify the ‘bad’ relations for the first two generations and/or modify the proton decay predictions [29,32] may also modify m_b/m_{τ} . In the models discussed below, the ratio of h_b/h_{τ} is similar to that found in the CMSSM. Here, we typically find that $h_b/h_{\tau} \simeq 0.65 - 0.75$ for $\tan \beta = 10$, and somewhat lower values $h_b/h_{\tau} \simeq 0.5 - 0.6$ for $\tan \beta = 55$. It is not possible in these models to force Yukawa coupling unification by choosing a suitable input value of h_b/h_{τ} [33]. Therefore, we use the following matching condition for the down-sector Yukawa couplings:

$$(h_b + h_{\tau})/2 = h_{\bar{\mathbf{5}}}/\sqrt{2}. \quad (14)$$

The matching conditions for the A terms are the same as those of the corresponding Yukawa couplings.

²Our sign convention for the A terms is the same as in Ref. [2,3,22], but opposite from that in Ref. [30] and in the ISAJET interface [31].

³ Because of different assumptions, our results cannot be compared directly with those of Ref.[22], though there are similarities.

We use for our calculations the program **SSARD** [34], which allows the computation of sparticle spectrum on the basis of 2-loop RGE evolution for the MSSM and 1-loop evolution for minimal SU(5). We define M_{GUT} as the scale where $g_1 = g_2$, so $M_{GUT} \simeq 1.5 \times 10^{16}$ GeV, but its exact value depends on the location in the parameter space. We also performed cross-checks with **ISAJET 7.80** [31] modified to include SU(5) running above M_{GUT} , and found results in very good agreement ⁴.

As a first illustration of the effects of the RGEs between M_{in} and M_{GUT} , we show in the top panels of Fig. 1 a comparison between the running of the soft masses: we plot $sign(m_i^2)\sqrt{|m_i^2|}$ for scalars and M_j for gauginos. Solid lines represent the CMSSM and dashed lines the SU(5) model specified at $M_{in} = 2.4 \times 10^{18}$ GeV with the same $\tan\beta = 10$ and $m_{1/2} = 900$ GeV, $m_0 = 218$ GeV and $A_0 = 0$. This point is chosen because it lies near the tip of the WMAP coannihilation strip for $\tan\beta = 10$ in the CMSSM and is similar to benchmark point H of [35]. There is some quantitative change in the evolution of gaugino masses M_i , but their qualitative behaviour is similar in the two models. On the other hand, the Higgs mass-squared parameters are significantly renormalized by g_5 , since the $m_{1/2}$ gauge-gaugino term dominates in the SU(5) RGEs. In the case of SU(5), the large value of M_5 initially causes \mathcal{H}_1 and \mathcal{H}_2 masses to evolve upward, but ultimately the λ^2 term takes over and reverses the direction of evolution. As a result, the $\mathcal{H}_{1,2}$ masses at M_{GUT} are not too large compared to m_0^2 , and $m_{\mathcal{H}_1}^2 > m_{\mathcal{H}_2}^2$ due to the relative strength of the $h_{\mathbf{5}}^2$ and $h_{\mathbf{10}}^2$ terms. The running of $m_{\mathcal{H}_1}^2$ and $m_{\mathcal{H}_2}^2$ between M_{in} and M_{GUT} evidently yields $m_{H_u}^2 \neq m_{H_d}^2 \neq m_0^2$ at the GUT scale. This is equivalent to the structure found in non-universal Higgs mass models (NUHM) [36, 37]. As a result, the following combination of soft supersymmetry-breaking parameters [38]:

$$S \equiv m_{H_u}^2 - m_{H_d}^2 + 2(m_{Q_1}^2 - m_{L_1}^2 - 2m_{U_1}^2 + m_{D_1}^2 + m_{E_1}^2) + (m_{Q_3}^2 - m_{L_3}^2 - 2m_{U_3}^2 + m_{D_3}^2 + m_{E_3}^2), \quad (15)$$

which vanishes at the GUT scale in the CMSSM, is non-vanishing here and affects the running of most of the soft mass parameters below M_{GUT} . The behaviour of $m_{H_u}^2$ (green lines) and $m_{H_d}^2$ (red lines) in Fig. 1 is qualitatively similar in the two models, but weak-scale values are larger in magnitude in SU(5) due to the larger GUT-scale values. We emphasize that EWSB occurs not when $m_{H_u}^2$ turns negative, but rather when $m_{H_u}^2 + \mu^2$ does, because it is the latter combination that appears in the Higgs potential that develops a minimum (see *e.g.* [2, 3]).

Sfermion masses are renormalized significantly more above M_{GUT} , and we notice big changes in the behaviours of $\tilde{\tau}_R$ (magenta lines) and \tilde{e}_R (tan lines). Their masses at the electroweak scale are much larger in the model with $M_{in} = 2.4 \times 10^{18}$ GeV than in the CMSSM. This reflects the large renormalization of the soft supersymmetry-breaking masses of all the **10** sfermions by gauge interactions between M_{in} and M_{GUT} . On the other hand, the $\tilde{\tau}_L$ (blue lines) and other $\mathbf{\bar{5}}$ sfermions are slightly less renormalized between M_{in} and M_{GUT} than the **10**'s, so that the $\tilde{L} - \tilde{E}^c$ mass difference is smaller than in the CMSSM. We also note the split of the values of the third-generation sfermion masses from those of

⁴For a comparison of the **SSARD** and **ISAJET** codes, see Ref. [35].

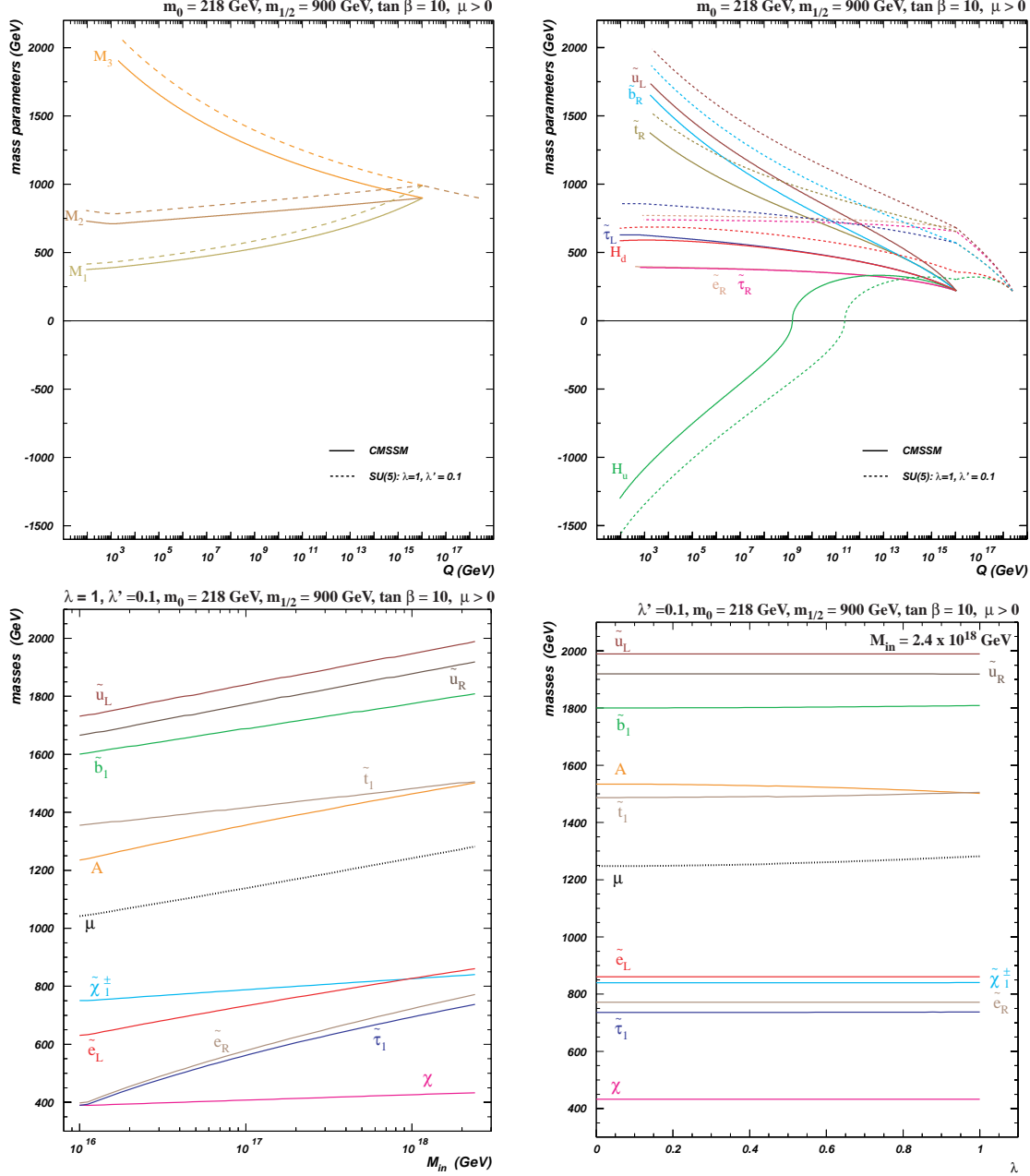


Figure 1: For a coannihilation point with $m_0 = 218 \text{ GeV}$, $m_{1/2} = 900 \text{ GeV}$, $A_0 = 0$, $\tan \beta = 10$, and $\lambda' = 0.1$. Top Left: the evolutions of the gaugino mass parameters with the choices $M_{in} = M_{GUT}$ (solid lines) and $\overline{M}_P = 2.4 \times 10^{18} \text{ GeV}$ (dashed lines), assuming $\lambda = 1$; Top Right: the evolutions of the soft supersymmetry-breaking scalar mass parameters with $M_{in} = M_{GUT}$ (solid lines) and \overline{M}_P (dashed lines), assuming $\lambda = 1$; Bottom Left: the dependences of the physical sparticle/Higgs masses on M_{in} assuming $\lambda = 1$; Bottom Right: the dependences of the sparticle/Higgs masses on λ assuming $M_{in} = \overline{M}_P$.

the first two generations in the SU(5) model: it is most noticeable in the $\tilde{\tau}_R - \tilde{e}_R$ mass difference. As mentioned in Ref. [30], this effect is due to the large Yukawa couplings of the third generation, and can be up to $\sim 20\%$. For this reason, our analysis is not entirely equivalent to SU(5)-inspired studies (see *e.g.* Ref. [39, 40]) where $m_{\tilde{\mathbf{5}}}$, $m_{\mathbf{10}}$ ($= m_{\tilde{\mathbf{5}},1}$, $m_{\mathbf{10},1}$) and $m_{\mathcal{H}_1, \mathcal{H}_2}$ are treated as free parameters at M_{GUT} .

The relatively large renormalization of the $\tilde{\tau}_R$ mass between M_{in} and M_{GUT} has some important implications for the appearances of the $(m_{1/2}, m_0)$ planes for large M_{in} . One, as we discuss in more detail below, is that the boundary where the LSP becomes the $\tilde{\tau}_1$ recedes to lower $m_{1/2}$ and m_0 , since only for smaller values of these parameters is the super-GUT renormalization insufficient to maintain $m_{\tilde{\tau}_1} > m_\chi$. As a consequence, the coannihilation strip also recedes to lower $m_{1/2}$ and m_0 , since it is only close to the $\tilde{\tau}_1$ LSP boundary that $m_{\tilde{\tau}_1} - m_\chi$ is small enough for coannihilation to bring the relic density down into the WMAP range⁵.

The bottom left panel of Fig. 1 shows how the sparticle spectrum evolves as a function of M_{in} between M_{GUT} and \overline{M}_P , for the same $\tan\beta = 10$, $m_{1/2} = 900$ GeV, $m_0 = 218$ GeV and $A_0 = 0$ as in the left panel. Note the rapid monotonic increase in $m_{\tilde{\tau}_1} - m_\chi$ with M_{in} . Since the change in $m_{\tilde{\tau}_R}^2$ is largely independent of m_0 , depending only on $m_{1/2}$, it is clear that, as M_{in} increases, $m_{\tilde{\tau}_1} < m_\chi$ only for ever-smaller values of $m_{1/2}$ and m_0 . Likewise, only for ever-smaller values of $m_{1/2}$ and m_0 is $m_{\tilde{\tau}_1} - m_\chi$ small enough for coannihilation to bring the relic density into the WMAP range.

The effect of super-GUT renormalization of the masses of $\tilde{\tau}_1$ and χ is illustrated in more detail in Fig. 2, which shows the ratio $m_\chi/m_{\tilde{\tau}_1}$ as a function of $m_{1/2}$ for fixed $m_0 = 300$ GeV and different values of M_{in} . Whilst the ratio $m_\chi/m_{\tilde{\tau}_1}$ increases as a function of $m_{1/2}$ in each case, its slope with respect to $m_{1/2}$ is sensitive to both M_{in} and $\tan\beta$. For low values of $m_{1/2}$, the ratio is relatively independent of M_{in} but sensitive to $\tan\beta$. However, for larger values of $m_{1/2}$ the super-GUT renormalization depends strongly on M_{in} , and the $\tilde{\tau}_1$ receives a larger super-GUT contribution to its mass than does χ , suppressing $m_\chi/m_{\tilde{\tau}_1}$. The green shaded horizontal strip highlights the area where $0.9 \leq m_\chi/m_{\tilde{\tau}_1} \leq 1.0$, *i.e.*, the region where stau-coannihilation is expected to be important [17]. When $M_{in} = M_{GUT}$, $m_\chi/m_{\tilde{\tau}_1} > 1$ at $m_{1/2} = 1180$ GeV for $\tan\beta = 10$, and coannihilation is important only for a small range of slightly larger values of M_{in} . However, the ratio $m_\chi/m_{\tilde{\tau}_1}$ never exceeds 0.8 for $M_{in} > 10^{17}$ GeV, and $\tilde{\tau}$ coannihilation is not effective for any value of $m_{1/2}$. A similar trend is seen for $\tan\beta = 55$, though larger M_{in} is needed before coannihilation ceases to be effective.

The bottom right panel of Fig. 1 shows the sensitivities to the value of λ of the results for the point $m_0 = 300$ GeV, $A_0 = 0$ and $\tan\beta = 10$. We see that they are quite weakly sensitive for this point, that was chosen in the coannihilation strip region. As we mentioned earlier, RGE evolution for this point is dominated by gauge-gaugino terms, so a large λ can only affect Higgs sector soft masses and consequently m_A and μ , which are not relevant for neutralino annihilation in this case.

⁵In minimal SU(5) with unification, just as in the CMSSM, the $\tilde{\tau}_1$ is dominantly $\tilde{\tau}_R$. Coannihilation with $\tilde{\tau}_1 \simeq \tilde{\tau}_L$, as found in Ref. [39] in a SU(5)-inspired framework, requires $m_{\mathbf{10}}/m_{\tilde{\mathbf{5}}} \simeq 50$ at M_{GUT} , which cannot be realized in the unified framework studied here.

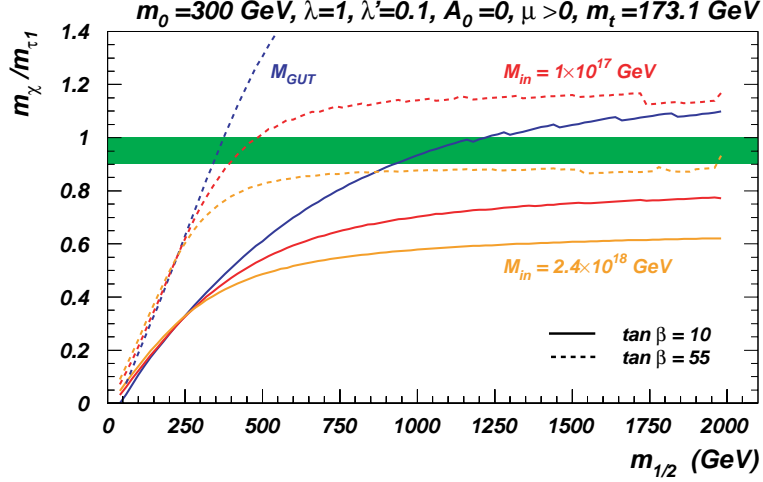


Figure 2: The ratio neutralino to stau masses as a function of $m_{1/2}$ for three choices of $M_{in} = M_{GUT}, 10^{17} \text{ GeV}$ and \overline{M}_P for the choices $m_0 = 300 \text{ GeV}$, $A_0 = 0$ and $\tan \beta = 10$ (solid) or 55 (dashed), assuming $\lambda = 1, \lambda' = 0.1$. The shaded green horizontal band highlights the regime in which stau coannihilation is important.

Fig. 3 shows a similar set of panels for a typical focus point with $m_0 = 2005 \text{ GeV}$, $m_{1/2} = 300 \text{ GeV}$, $A_0 = 0$ and $\tan \beta = 10$ (equivalent to benchmark point E [35]). The upper right panel (for $\lambda = 1$) shows that in this case the most dramatic effects of increasing M_{in} are on $m_{H_u}^2$ (green lines) and $m_{H_d}^2$ (red lines). Since $m_{1/2} \ll m_0$, gauge-gaugino terms play only a subdominant role in the evolutions of these masses-squared. Therefore, if universality is assumed at \overline{M}_P (dashed lines), they are both renormalized significantly by λ as Q decreases from \overline{M}_P to M_{GUT} . Furthermore, at M_{GUT} we have $m_{H_u}^2 < m_{H_d}^2$ and hence $S \simeq m_{H_u}^2 - m_{H_d}^2 < 0$ (neglecting here insignificant contributions from the squark mass splittings). (We recall that S (15) enters into the MSSM RGEs for $m_{H_u}^2$ and $m_{H_d}^2$, but is equal to 0 in the CMSSM.) The effect of $S < 0$ on the running of the Higgs soft masses is to push $|m_{H_u}^2|$ to higher values, resulting in the larger value of μ parameter⁶. This increase in the value of μ suggests that the boundary of electroweak symmetry breaking recedes to larger m_0 for large λ , as we confirm later. We also see that the third-generation tenplet mass decreases significantly, due to the effect of the large h_{10}^2 term.

The bottom left panel of Fig. 3 shows that the low-scale sparticle masses are relatively insensitive to M_{in} , though $m_{\tilde{\tau}_1}, m_A$ and $m_{\tilde{t}_1}$ do indicate some sensitivity. As noted above, there is a rapid increase in μ as M_{in} increases above M_{GUT} . In the focus-point regime [20], the LSP has significant higgsino component so its mass $\simeq \mu$. As M_{in} is increased, the LSP mass is once again $\simeq M_1$ and this implies that the relic density is larger than that allowed by astrophysics and cosmology if m_0 is kept fixed.

The bottom right panel of Fig. 3 shows how these effects depend on the choice of λ . We note in particular that μ increases significantly even for $\lambda \sim 0.3$. This suggests that

⁶As a additional consequence, the scale at which $m_{H_u}^2$ is driven negative is much larger than in the standard CMSSM $M_{in} = M_{GUT}$ case (solid lines).

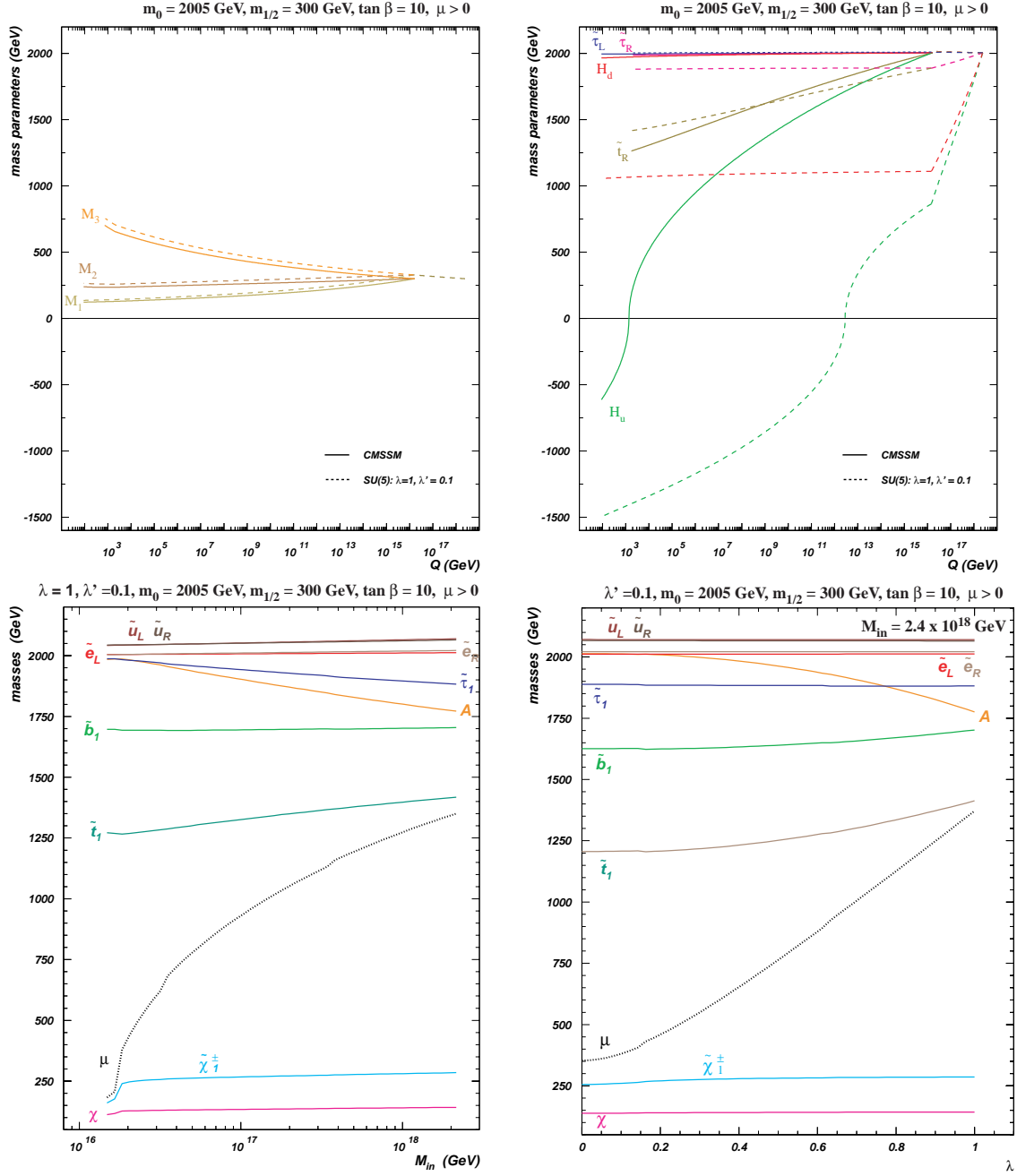


Figure 3: As for Fig. 1, but for a focus point with $m_0 = 2005$ GeV, $m_{1/2} = 300$ GeV, $A_0 = 0$ and $\tan \beta = 10$.

the effects of λ on the locations of the focus-point region and the boundary of electroweak symmetry breaking sets in already for moderate values of λ . We also see that the $\tilde{\tau}_1$ mass is insensitive to λ . Since $m_{\tilde{\tau}_R}^2$ is not significantly renormalized below M_{GUT} , we can confirm that the running of the third-generation **10**-plet mass above M_{GUT} is dominated by the large h_{10}^2 term. The observed sensitivities of the stop masses are entirely due to the influence on MSSM running of changing boundary conditions at M_{GUT} .

Fig. 4 shows a similar analysis for a point in the rapid-annihilation funnel region [8] with $m_0 = 1700$ GeV, $m_{1/2} = 1500$ GeV, $A_0 = 0$ and $\tan \beta = 55$ (similar to benchmark point M [35]). We see in the top left panel that the effects of the renormalization between \overline{M}_P and M_{GUT} are important for $m_{H_u}^2$ (green lines) and $m_{H_d}^2$ (red lines), and somewhat smaller for most sparticle masses. In this region, the relic density is mainly controlled by the relation between m_A and m_χ . We see in the bottom right panel of Fig. 4 that these vary in similar ways for $M_{GUT} < M_{in} < \overline{M}_P$, suggesting that the location of the rapid-annihilation funnel may be relatively insensitive to M_{in} . Finally, we see in the bottom right panel of Fig. 4 that most of the sparticle masses are relatively insensitive to λ . The exception is $m_{\tilde{\tau}_1}$, which is, however, of little relevance to the relic density in this region of the $(m_{1/2}, m_0)$ plane. We see again that the relation between m_A and m_χ is quite stable, exhibiting little sensitivity to λ .

3 Representative $(m_{1/2}, m_0)$ Planes

We now discuss the consequences of the additional RGE evolution between M_{in} and M_{GUT} for the CMSSM parameter space for some representative $(m_{1/2}, m_0)$ planes for fixed $\tan \beta$ and A_0 . In order to span the range of plausible values of $\tan \beta$, we display planes for $\tan \beta = 10$ and 55. We consider values of M_{in} up to $\overline{M}_P \equiv M_P/\sqrt{2\pi} \sim 2.4 \times 10^{18}$ GeV, but restrict our attention to $A_0 = 0$. We assume $m_t = 173.1$ GeV [41] and $m_b^{\overline{MS}}(m_b) = 4.2$ GeV [42]. In all these $(m_{1/2}, m_0)$ planes, we indicate by brown shading the regions that are excluded because the LSP is the lighter $\tilde{\tau}$. Regions where consistent vacuum breaking electroweak symmetry does not occur are indicated by orange shading. We compute $BR(b \rightarrow s\gamma)$ as in [43], and treat errors according to the procedure outlined in Ref. [44] when studying the compatibility with the experimentally measured value [45]; regions excluded at 95%CL are shaded green. Regions favoured by $g_\mu - 2$ measurements [46, 47] if the Standard Model contribution is calculated using low-energy e^+e^- data [48] are shaded pink, with the $\pm 1\text{-}\sigma$ contours shown as black dashed lines and the $\pm 2\text{-}\sigma$ contours shown as solid black lines. We restrict our attention to $\mu > 0$, as suggested by both $b \rightarrow s\gamma$ and $g_\mu - 2$. The LEP lower limit on the chargino mass [19] is shown as a thick black dashed line, and the LEP lower limit on m_h [49] is indicated as a red dash-dotted line which shows the position of the $m_h = 114.4$ GeV contour. The light Higgs mass is computed using the `FeynHiggs 2.6.5` code [50], and recall that its nominal results should be assigned a theoretical error ~ 1.5 GeV, so that the location of the constraint contour is only approximate. Finally, we use blue colour to indicate the regions where the neutralino relic density falls within the $2\text{-}\sigma$ WMAP range [16], $0.097 \leq \Omega_{CDM}h^2 \leq 0.122$.

Fig. 5 displays representative $(m_{1/2}, m_0)$ planes for $\tan \beta = 10$ and various values of M_{in}

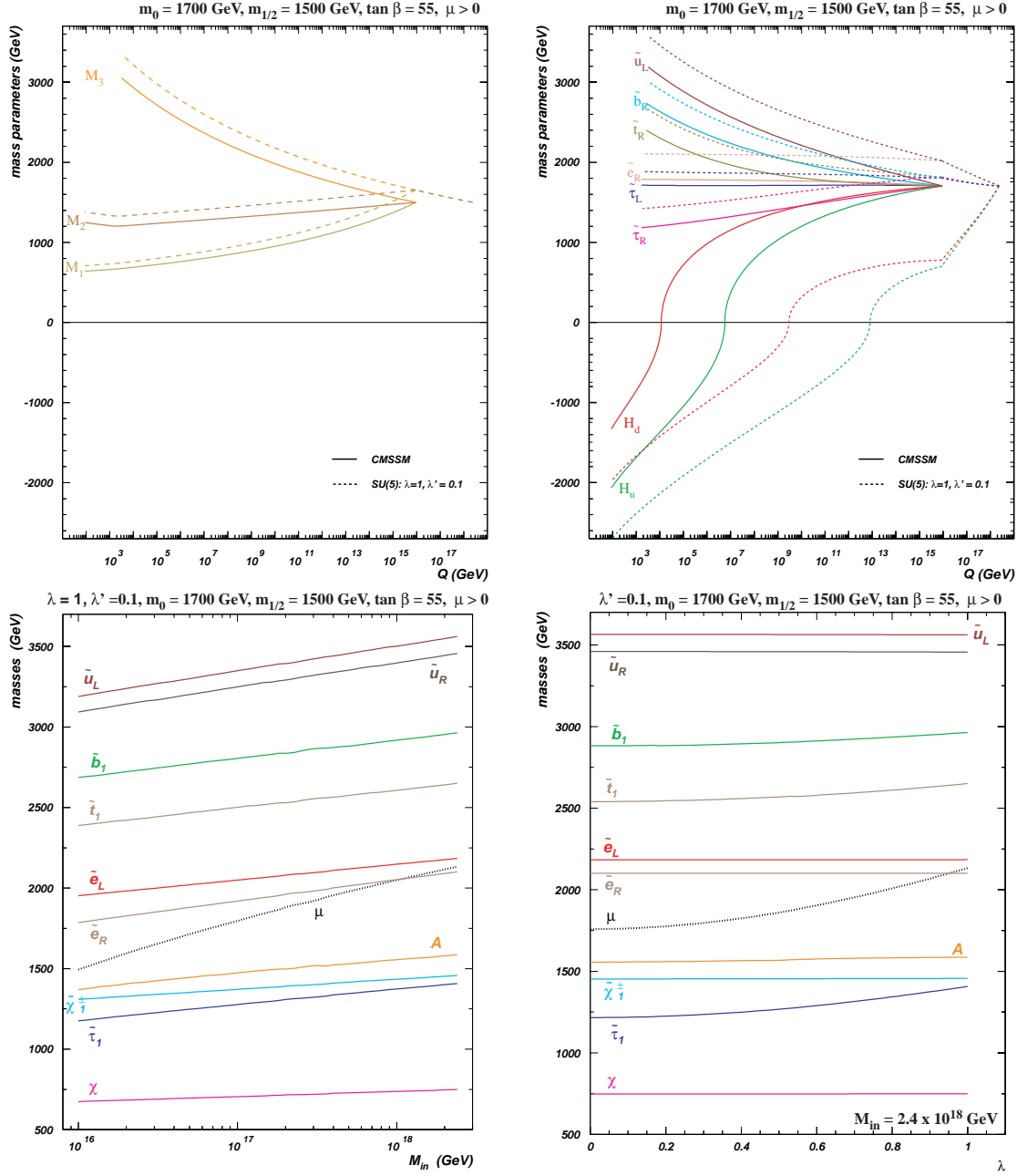


Figure 4: As for Fig. 1, but for a rapid-annihilation funnel point with $m_0 = 1700$ GeV, $m_{1/2} = 1500$ GeV, $A_0 = 0$ and $\tan \beta = 55$.

and the specific choices $\lambda = 1, \lambda' = 0.1$. The choice of a relatively large value of λ highlights its potential importance (a smaller value $\lambda = 0.1$ is discussed later), and our results are quite insensitive to the value of λ' . The choices of λ and λ' are irrelevant for the choice $M_{in} = M_{GUT}$ shown in the top left panel of Fig. 5. Here, we see the familiar features of a stau coannihilation strip extending up to $m_{1/2} \sim 900$ GeV, close to the boundary with the stau LSP region, and a focus-point strip close to the boundary of electroweak symmetry breaking. There is some tension between the m_h and $g_\mu - 2$ constraints, but a region of the coannihilation strip with $m_{1/2} \sim 400$ GeV would be consistent with both constraints.

Turning now to the choice $M_{in} = 2.5 \times 10^{16}$ GeV, shown in the top right panel of Fig. 5, we see two dramatic effects from the modest increase in M_{in} . One is the rapid disappearance of the stau LSP region, which has retreated to $m_0^2 < 0$ ⁷, as a direct result of the renormalization effects seen in the left panel of Fig. 1. A similar effect was seen in [23, 24]. Because the ratio $m_\chi/m_{\tilde{\tau}}$ is reduced as M_{in} is increased, as shown in Fig. 2, there is a corresponding shift in the coannihilation strip to smaller m_0 and $m_{1/2}$. In the particular example shown, the coannihilation strip extends to $m_{1/2} \sim 450$ GeV, and there is a healthy portion compatible with the $g_\mu - 2$ constraint⁸. We note, in particular, that there is a region with $m_0 = 0$ [52, 53] which is compatible with all the constraints for $360 \text{ GeV} \lesssim m_{1/2} \lesssim 450 \text{ GeV}$: however, its existence is very sensitive to the choice of M_{in} . The other noticeable feature (also seen in [23]) of the top right panel of Fig. 5 is the retreat of the electroweak symmetry breaking constraint to smaller $m_{1/2}$ and larger m_0 . As we discuss later, this effect is quite sensitive to the value of λ , whereas the fate of the coannihilation region is relatively insensitive to its value.

For the choice $M_{in} = 10^{17}$ GeV, shown in the bottom left panel of Fig. 5, these effects are more pronounced: both the coannihilation and the focus-point strips have disappeared entirely. There is a small piece of the $(m_{1/2}, m_0)$ plane where the relic density falls within the WMAP range, but this is incompatible with m_h and gives too large a value of $g_\mu - 2$. Finally, for the choice $M_{in} = 2.4 \times 10^{18}$ GeV, shown in the bottom right panel of Fig. 5, the small residual WMAP region falls foul also of the chargino mass constraint.

We now consider the choice $\tan \beta = 55$, shown in Fig. 6. In this case, in addition to the coannihilation and focus-point strips seen previously, we also note the rapid-annihilation funnel that appears for $1000 \text{ GeV} \lesssim m_{1/2} \lesssim 1500 \text{ GeV}$. As M_{in} increases, the renormalization effects seen in the left panel of Fig. 1 cause the stau LSP region to retreat as in the $\tan \beta = 10$ case, though more slowly, and it does not disappear entirely, even for $M_{in} = 2.4 \times 10^{18}$ GeV. Likewise, whilst the coannihilation strip shrinks with increasing M_{in} , it does not disappear, and much of it remains consistent with m_h , $b \rightarrow s\gamma$ and $g_\mu - 2$. The rapid-annihilation funnel also persists as M_{in} increases, staying in a similar range of $m_{1/2}$, but shifting gradually to lower values of m_0 . In particular, we note that for the case $M_{in} = 2.4 \times 10^{18}$ GeV, the no-scale possibility $m_0 = 0$ [52, 53] is allowed, on one or both flanks of the rapid-annihilation funnel. Finally, we note that the electroweak symmetry breaking boundary disappears entirely for the displayed choices of $M_{in} > M_{GUT}$, as does the focus-point WMAP strip.

⁷See [51] for a discussion of the circumstances under which this might be cosmologically acceptable.

⁸The chargino, m_h , $g_\mu - 2$ and $b \rightarrow s\gamma$ constraints are relatively stable in the $(m_{1/2}, m_0)$ plane with respect to changes in M_{in} .

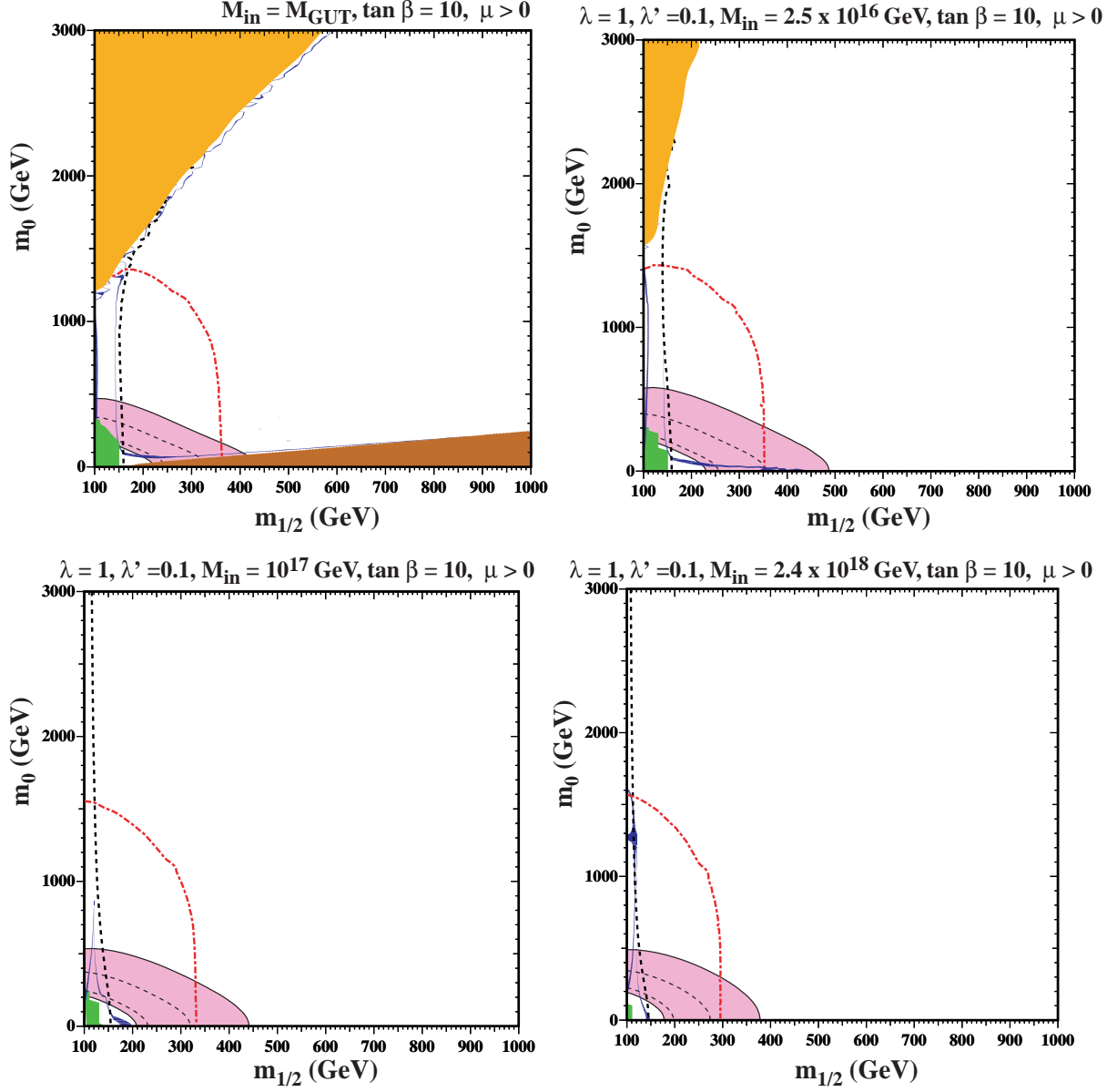


Figure 5: The $(m_{1/2}, m_0)$ planes for $A_0 = 0, \tan \beta = 10, \lambda = 1$ and $\lambda' = 0.1$ for different choices of M_{in} : M_{GUT} (top left), 2.5×10^{16} GeV (top right), 10^{17} GeV (bottom left), and 2.4×10^{18} GeV (bottom right). In the blue regions, $\Omega_\chi h^2$ is within the WMAP range. Pink region between black dashed (solid) lines are allowed by $g_\mu - 2$ at $1\text{-}\sigma$ ($2\text{-}\sigma$). Dark orange, brown and green colored regions are excluded. Areas to the left of thick black dashed and red dash-dotted lines are ruled out by LEP searches for charginos and light MSSM Higgs respectively. More details can be found in the text.

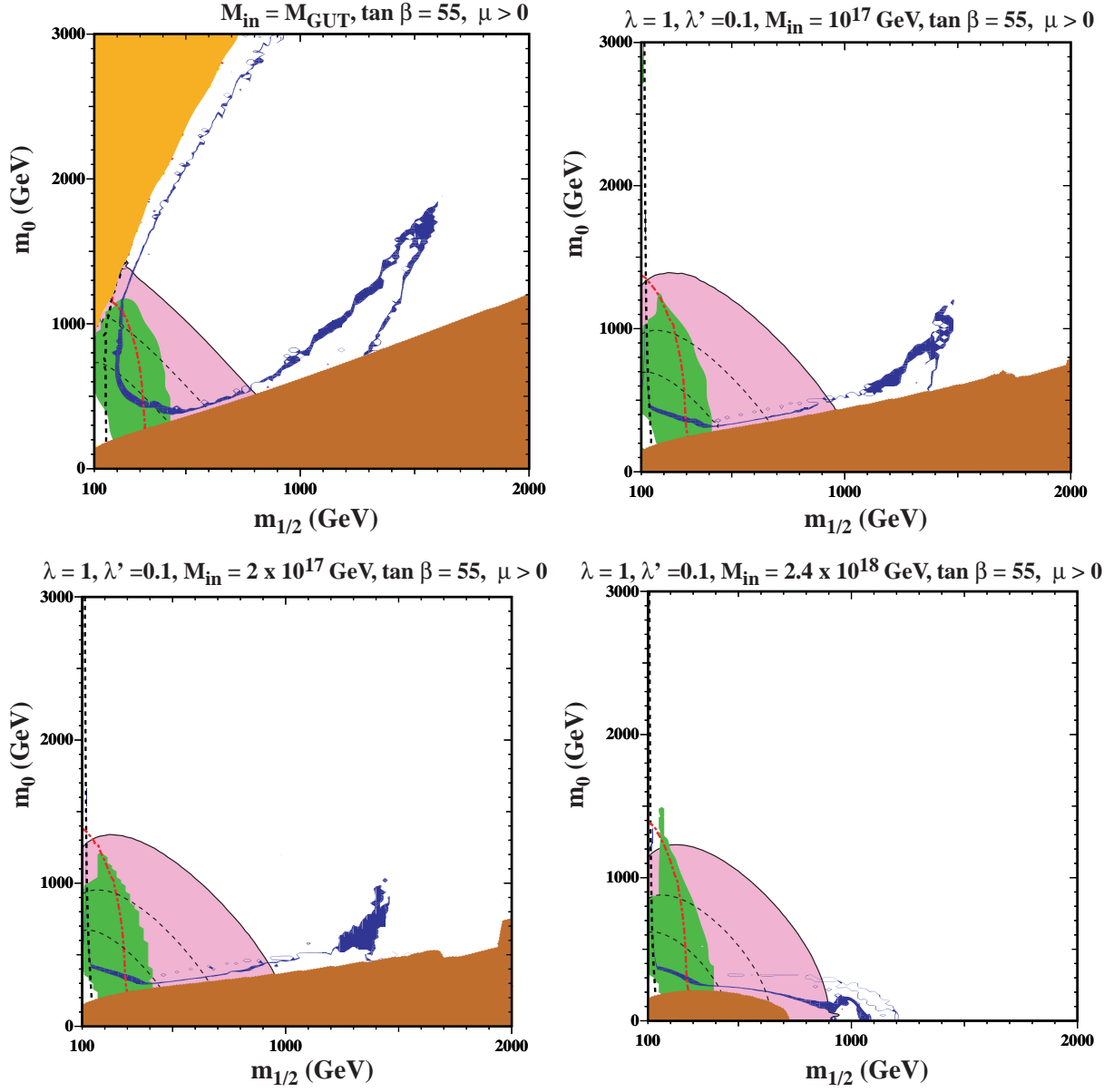


Figure 6: As for Fig. 5, for $\tan \beta = 55$, $\lambda = 1$ and $\lambda' = 0.1$ for different choices of M_{in} : M_{GUT} (top left), 10^{17} GeV (top right), 2×10^{17} GeV (bottom left), and 2.4×10^{18} GeV (bottom right).

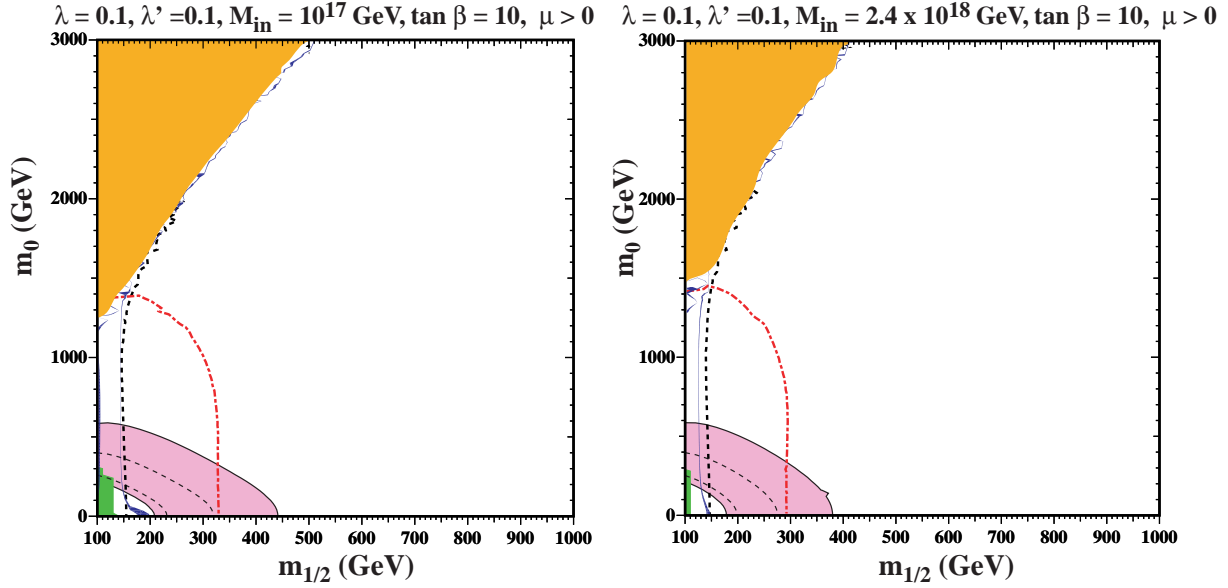


Figure 7: As for Fig. 5, for $\tan \beta = 10$, $\lambda = 0.1$ and $\lambda' = 0.1$ for different choices of M_{in} : 10^{17} GeV (left) and 2.4×10^{18} GeV (right).

In order to see the importance of the choice of λ , we display in Fig. 7 two examples of $(m_{1/2}, m_0)$ planes for the choice $\lambda = 0.1$ and $\tan \beta = 10$, assuming $M_{in} = 10^{17}$ GeV and 2.4×10^{18} GeV. Comparing with the bottom panels of Fig. 5, we see very little change in the low- m_0 parts of the planes: in particular, the stau LSP region and the coannihilation strip have disappeared in the same ways. On the other hand, we see at large m_0 that the electroweak symmetry breaking boundary is very similar for the two choices $M_{in} = 10^{17}$ GeV and 2.4×10^{18} GeV, and that these are in turn very similar to the corresponding boundary for $M_{in} = M_{GUT}$, shown in the top left panel of Fig. 5. This confirms that the differences in the focus-point regions shown in the other panels of Fig. 5 are due to the choice $\lambda = 1$ made there.

Fig. 8 shows a similar pair of comparisons for $\tan \beta = 55$, with the choice $\lambda = 0.1$ and $M_{in} = 10^{17}$ GeV, 2.4×10^{18} GeV. Comparing with the corresponding (right) panels of Fig. 6, we see that the stau LSP regions and the coannihilation strips are essentially identical, indicating that the value of λ is irrelevant in these regions, as expected. However, differences again are found at large m_0 , where the focus point region has made a reappearance where previously, it had disappeared for all values of $M_{in} > M_{GUT}$ when $\lambda = 1$, but is not only present for $\lambda = 0.1$, but has even moved to smaller values of m_0 than in the CMSSM case.

The structure of the relic density regions when $\tan \beta = 55$ is quite sensitive to our assumption for A_0 , as is also the case in the CMSSM, where the funnel regions become more pronounced when $A_0 < 0$ [54]. In Fig. 9, we illustrate this effect by taking $A_0 = -1.5m_{1/2}$ for $M_{in} = 10^{17}$ GeV (left), 2.4×10^{18} GeV (right) and $\lambda = 1, 0.1$ (upper and lower, respectively). In each case, when $A_0 < 0$ the funnel region reaches to higher values of m_0 and $m_{1/2}$ and its two branches are more clearly separated. When $\lambda = 0.1$, we also see that the rapid-

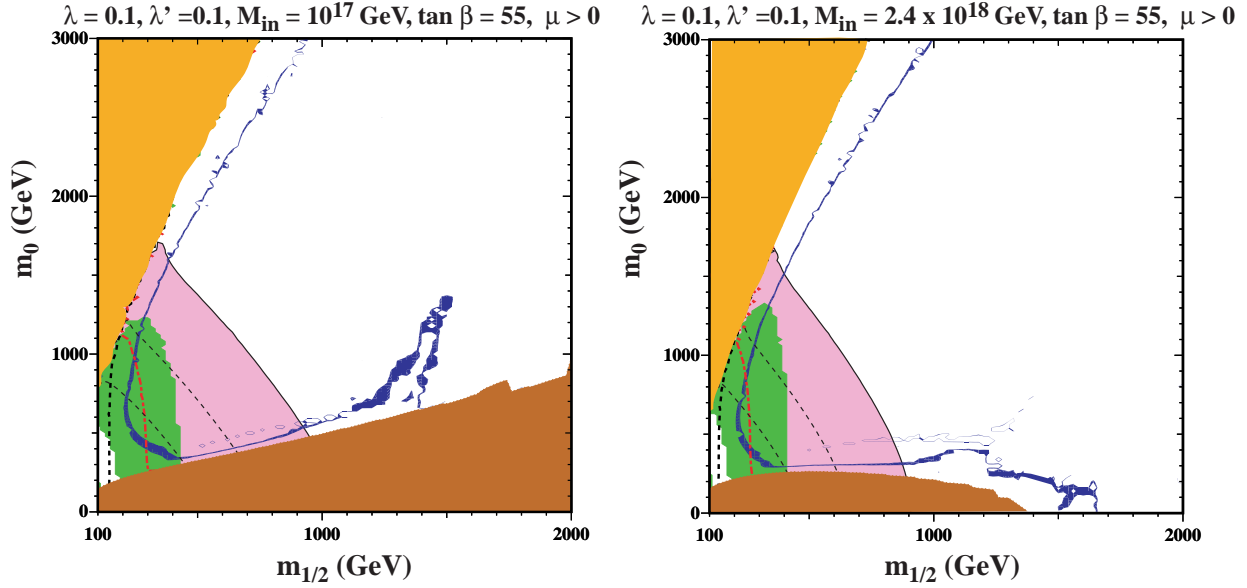


Figure 8: As for Fig. 7, for $A_0 = 0$, $\tan \beta = 55$, $\lambda = 0.1$ and $\lambda' = 0.1$ for different choices of M_{in} : 10^{17} GeV (left) and 2.4×10^{18} GeV (right).

annihilation funnel region begins to connect with the focus-point region.

An example of how the value of m_0 at the boundary of electroweak symmetry breaking depends on M_{in} for different values of λ is shown in the left panel of Fig. 10, where we have chosen $m_{1/2} = 300$ GeV, $A_0 = 0$. We see from the solid curves that when $\tan \beta = 10$ the EWSB boundary value of m_0 increases monotonically with M_{in} for any value of λ , and that the rate of increase itself grows with the value of λ . This is the result of the μ parameter getting larger (for fixed m_0) with M_{in} , as we discussed in the previous Section. However, for $\tan \beta = 55$, shown by the dashed curves, for small λ the EWSB boundary migrates to smaller m_0 as M_{in} increases. For such large values of $\tan \beta$, $h_{\overline{5},10} \gg h_{10}$, producing a stronger downward push in the evolution of $m_{\mathcal{H}_1}$ relative to that of $m_{\mathcal{H}_2}$. If $\lambda \lesssim 0.3$, its effects in the RGE's are negligible compared to those of the $h_{\overline{5},10}$ Yukawa couplings, resulting in a hierarchy $m_{\mathcal{H}_1} < m_{\mathcal{H}_2}$ at M_{GUT} . In this case, we have $S > 0$ at M_{GUT} which in turn leads to smaller values of μ and hence a smaller value of m_0 at the electroweak symmetry breaking boundary. At the ‘critical’ value of $\lambda \simeq 0.3$, the evolutions of the \mathcal{H}_1 and \mathcal{H}_2 soft masses are almost identical, with the result that at M_{GUT} , $m_{\mathcal{H}_1} \simeq m_{\mathcal{H}_2} < m_0$, so that $S \approx 0$ as in the CMSSM. As a result, the weak-scale value of μ is nearly the same as in the CMSSM – the well-known focusing behaviour – and the location of the EWSB boundary remains stable with respect to M_{in} . Finally, for larger λ , $S < 0$ at M_{GUT} , the weak-scale value of μ is larger than in the CMSSM, and the EWSB boundary migrates to larger m_0 .

The effects of λ are seen explicitly in the right panel of Fig. 10, where we show the location of the EWSB boundary as a function of λ for different choices of M_{in} and two values of $\tan \beta$. We note that even for vanishing λ the EWSB boundary changes with M_{in} due to the additional running of the soft masses above M_{GUT} .

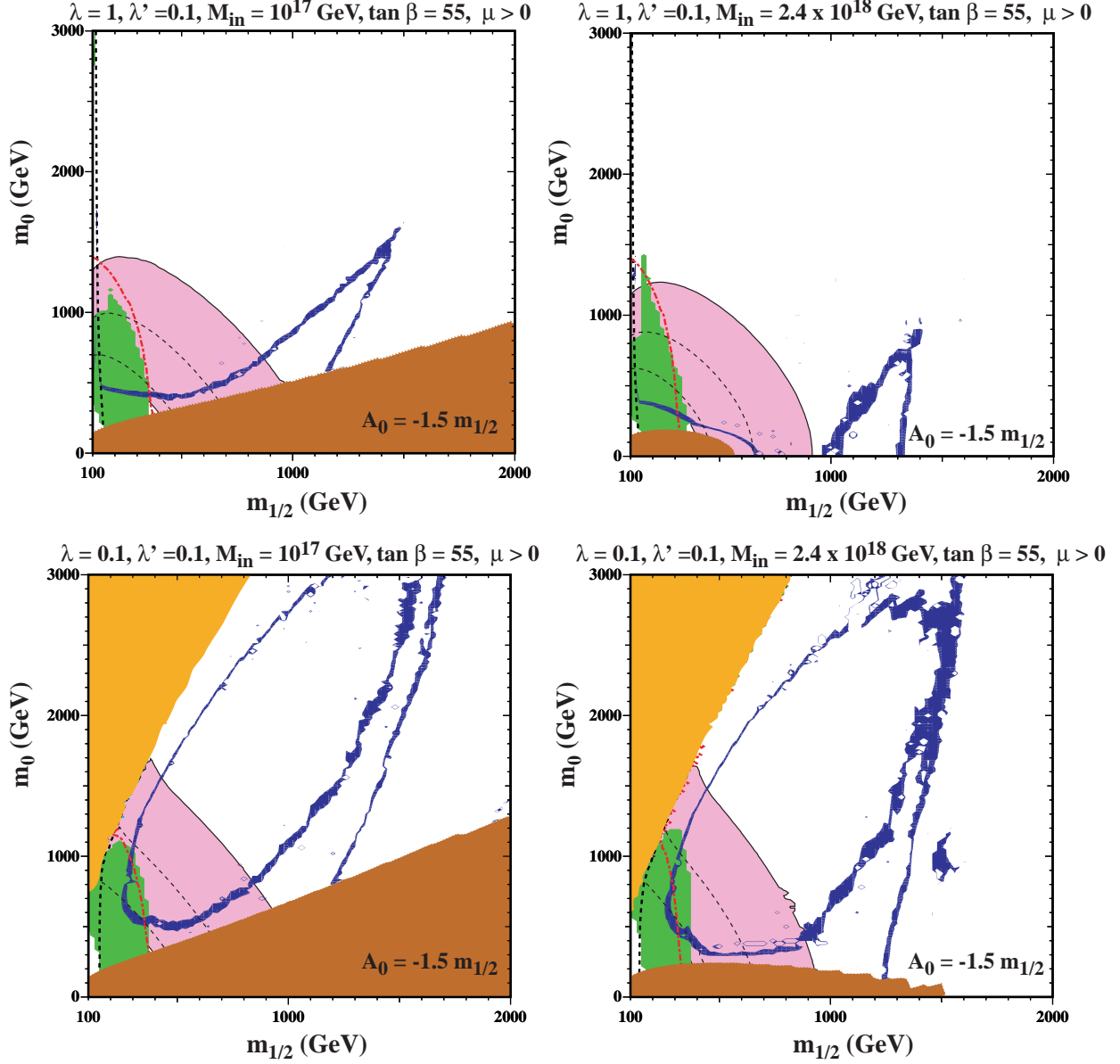


Figure 9: As for Fig. 5, for $\tan \beta = 55$, $A_0 = -1.5 m_{1/2}$, $\lambda' = 0.1$ for $\lambda = 1$ (upper) and $\lambda = 0.1$ (lower) with $M_{\text{in}} = 10^{17} \text{ GeV}$ (left), and $2.4 \times 10^{18} \text{ GeV}$ (right).

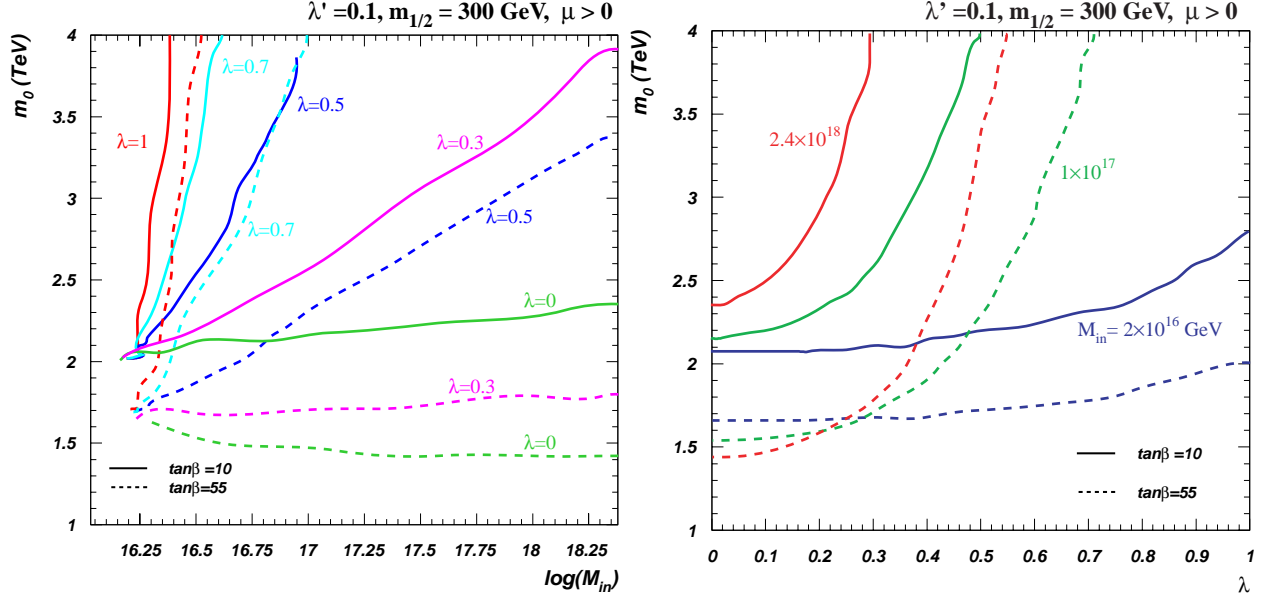


Figure 10: The value of m_0 at the boundary of electroweak symmetry breaking for $m_{1/2} = 300$ GeV, $A_0 = 0$ and $\tan\beta = 10$ (solid) and $=55$ (dashed), as a function of M_{in} for various values of λ (left panel) and as a function of λ for various M_{in} (right panel).

4 Summary of Results for Different $\tan\beta$

As we have discussed above, the $\tilde{\tau}$ coannihilation region tends to disappear as M_{in} increases, and the focus-point region tends to recede to larger m_0 as λ increases. So far, we have shown these effects only for $\tan\beta = 10$ and 55 . Here we summarize how these effects vary for intermediate values of $\tan\beta$.

We see in the left panel of Fig. 11 the region of the $(M_{in}, \tan\beta)$ plane where there is a coannihilation/rapid-annihilation strip. For choices lying below the contour, all the points where coannihilation or rapid-annihilation brings the relic density into the WMAP range are excluded by other constraints. For $\tan\beta \lesssim 20$, the region below the curve has m_h lower than the LEP bound, while for larger $\tan\beta$, $g_\mu - 2$ is too large in this region⁹. We see that, as $\tan\beta$ increases above 10, the coannihilation/rapid-annihilation strip persists up to progressively larger values of M_{in} . However, only for $\tan\beta > 47$ does it persist for $M_{in} = \overline{M}_P$. This information is potentially a useful diagnostic tool, if supersymmetry is discovered. For example, if experiments determine that m_0 is (essentially) universal and $m_0 \ll m_{1/2}$ with $\tan\beta \sim 20$, then we can infer from the left panel of Fig. 11 that $M_{in} < 10^{17}$ GeV.

We have also seen earlier that the focus-point region is sensitive to the value of λ . The right panel of Fig. 11 displays the regions of the (M_{in}, λ) plane where the focus-point strip has $m_0 < 5$ TeV for $m_{1/2} = 300$ GeV. We see immediately that these regions are rather similar

⁹The kink in the boundary contour is an artifact of our approximation to the m_h constraint: incorporating the uncertainty in the theoretical calculation of m_h and the experimental likelihood function would smooth it out. Recall that for very low $\tan\beta$, there is some tension between the Higgs mass and $g_\mu - 2$ constraints.

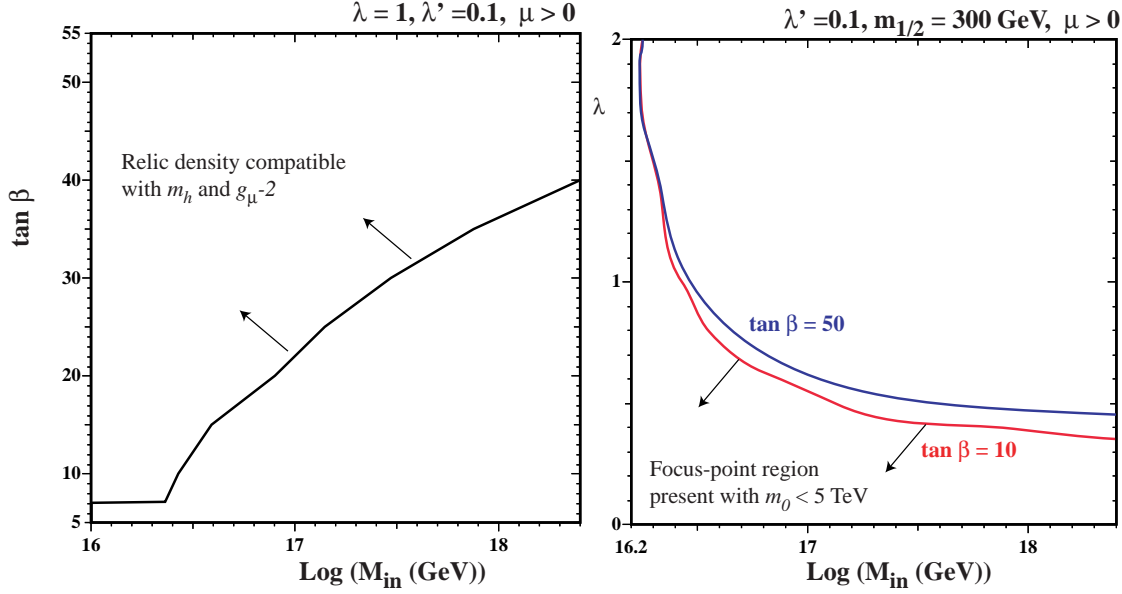


Figure 11: *Left: the coannihilation/rapid-annihilation strips are compatible with other experimental constraints only for values of $(M_{in}, \tan \beta)$ above the diagonal contour. Right: the focus-point strip has $m_0 < 5$ TeV at $m_{1/2} = 300$ GeV only for values of M_{in}, λ below the red (blue) line for $\tan \beta = 10(50)$.*

for $\tan \beta = 10$ and 50 , lying below the red and blue lines, respectively. This information could be used to infer a constraint on λ , which otherwise does not impact significantly low-energy phenomenology. For example, if experiment indicates that Nature is described by a focus-point model with $m_{1/2} = 300$ GeV, $m_0 < 5$ TeV and $M_{in} > 10^{17}$ GeV, then we can infer from the right panel of Fig. 11 that $\lambda < 0.6$.

5 Summary

We have shown in this paper that the characteristic $(m_{1/2}, m_0)$ planes of the CMSSM are significantly modified as M_{in} increases above M_{GUT} , and also depend on the SU(5) GUT Higgs coupling λ , whereas they are less sensitive to the other Higgs coupling λ' . Indeed, the familiar stau coannihilation strip and focus-point region may disappear to small $m_{1/2}$ and large m_0 , respectively, as M_{in} increases. These possibilities should be borne in mind when searching for supersymmetry at the LHC and elsewhere: if Nature turns out to choose either the stau coannihilation strip or the focus-point region, one may be able to derive an upper limit on M_{in} and derive a constraint on the GUT Higgs coupling λ .

These observations serve as another reminder that, although the CMSSM with its universal soft supersymmetry-breaking masses at the GUT scale is appealingly simple, even small modifications of its assumptions may change significantly the expected phenomenology. The CMSSM may be a comfortable base camp for exploring the landscape of supersymmetric phenomenology, but one must get out into the fresh air from time to time!

6 Acknowledgements

The work of A.M. and K.A.O. is supported in part by DOE grant DE-FG02-94ER-40823 at the University of Minnesota.

References

- [1] For a review see e.g. H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75.
- [2] H. Baer and X. Tata, *Weak Scale Supersymmetry: From Superfields to Scattering Events*, (Cambridge University Press, 2006).
- [3] M. Drees, R. Godbole and P. Roy, *Theory and phenomenology of sparticles: An account of four-dimensional $N=1$ supersymmetry in high energy physics*, (World Scientific, 2004).
- [4] J. R. Ellis and D. V. Nanopoulos, Phys. Lett. B **110**, 44 (1982).
- [5] For reviews, see: H. P. Nilles, Phys. Rep. **110** (1984) 1; A. Brignole, L. E. Ibanez and C. Munoz, arXiv:hep-ph/9707209.
- [6] R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B **119**, 343 (1982).
- [7] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B **573** (2003) 162 [arXiv:hep-ph/0305212], and Phys. Rev. D **70** (2004) 055005 [arXiv:hep-ph/0405110].
- [8] M. Drees and M. M. Nojiri, Phys. Rev. D **47** (1993) 376 [arXiv:hep-ph/9207234]; H. Baer and M. Brhlik, Phys. Rev. D **53** (1996) 597 [arXiv:hep-ph/9508321]; Phys. Rev. D **57** (1998) 567 [arXiv:hep-ph/9706509]; H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D **63** (2001) 015007 [arXiv:hep-ph/0005027]; A. B. Lahanas, D. V. Nanopoulos and V. C. Spanos, Mod. Phys. Lett. A **16** (2001) 1229 [arXiv:hep-ph/0009065]; A. B. Lahanas and V. C. Spanos, Eur. Phys. J. C **23** (2002) 185 [arXiv:hep-ph/0106345].
- [9] J. R. Ellis, T. Falk, K. A. Olive and M. Schmitt, Phys. Lett. B **388** (1996) 97 [arXiv:hep-ph/9607292]; Phys. Lett. B **413** (1997) 355 [arXiv:hep-ph/9705444]; J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Schmitt, Phys. Rev. D **58** (1998) 095002 [arXiv:hep-ph/9801445]; V. D. Barger and C. Kao, Phys. Rev. D **57** (1998) 3131 [arXiv:hep-ph/9704403]. J. R. Ellis, T. Falk, G. Ganis and K. A. Olive, Phys. Rev. D **62** (2000) 075010 [arXiv:hep-ph/0004169].
- [10] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B **510** (2001) 236 [arXiv:hep-ph/0102098].
- [11] V. D. Barger and C. Kao, Phys. Lett. B **518** (2001) 117 [arXiv:hep-ph/0106189]; L. Roszkowski, R. Ruiz de Austri and T. Nihei, JHEP **0108** (2001) 024 [arXiv:hep-ph/0106334]; A. Djouadi, M. Drees and J. L. Kneur, JHEP **0108** (2001) 055 [arXiv:hep-ph/0107316]; U. Chattopadhyay, A. Corsetti and P. Nath, Phys. Rev. D **66** (2002)

- 035003 [arXiv:hep-ph/0201001]; J. R. Ellis, K. A. Olive and Y. Santoso, *New Jour. Phys.* **4** (2002) 32 [arXiv:hep-ph/0202110]; H. Baer, C. Balazs, A. Belyaev, J. K. Mizukoshi, X. Tata and Y. Wang, *JHEP* **0207** (2002) 050 [arXiv:hep-ph/0205325]; R. L. Arnowitt and B. Dutta, arXiv:hep-ph/0211417.
- [12] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, *Phys. Lett. B* **565** (2003) 176 [arXiv:hep-ph/0303043]; H. Baer and C. Balazs, *JCAP* **0305**, 006 (2003) [arXiv:hep-ph/0303114]; A. B. Lahanas and D. V. Nanopoulos, *Phys. Lett. B* **568**, 55 (2003) [arXiv:hep-ph/0303130]; U. Chattopadhyay, A. Corsetti and P. Nath, *Phys. Rev. D* **68**, 035005 (2003) [arXiv:hep-ph/0303201]; C. Munoz, *Int. J. Mod. Phys. A* **19**, 3093 (2004) [arXiv:hep-ph/0309346].
- [13] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, *Phys. Rev. D* **69** (2004) 095004 [arXiv:hep-ph/0310356]; J. Ellis, S. Heinemeyer, K. Olive and G. Weiglein, *JHEP* **0502** 013, hep-ph/0411216; J. R. Ellis, S. Heinemeyer, K. A. Olive and G. Weiglein, *JHEP* **0605**, 005 (2006) [arXiv:hep-ph/0602220].
- [14] O. Buchmueller *et al.*, *Phys. Lett. B* **657** (2007) 87 [arXiv:0707.3447 [hep-ph]]; O. Buchmueller *et al.*, *JHEP* **0809** (2008) 117 [arXiv:0808.4128 [hep-ph]]; O. Buchmueller *et al.*, *Eur. Phys. J. C* **64**, 391 (2009) [arXiv:0907.5568 [hep-ph]].
- [15] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, *Nucl. Phys. B* **238** (1984) 453; see also H. Goldberg, *Phys. Rev. Lett.* **50** (1983) 1419.
- [16] J. Dunkley *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 306 (2009) [arXiv:0803.0586 [astro-ph]]; E. Komatsu *et al.* [WMAP Collaboration], *Astrophys. J. Suppl.* **180**, 330 (2009) [arXiv:0803.0547 [astro-ph]].
- [17] J. R. Ellis, T. Falk and K. A. Olive, *Phys. Lett. B* **444** (1998) 367 [arXiv:hep-ph/9810360]; J. R. Ellis, T. Falk, K. A. Olive and M. Srednicki, *Astropart. Phys.* **13** (2000) 181 [Erratum-ibid. **15** (2001) 413] [arXiv:hep-ph/9905481]; R. Arnowitt, B. Dutta and Y. Santoso, *Nucl. Phys. B* **606** (2001) 59 [arXiv:hep-ph/0102181]; M. E. Gómez, G. Lazarides and C. Pallis, *Phys. Rev. D* **D61** (2000) 123512 [arXiv:hep-ph/9907261]; *Phys. Lett.* **B487** (2000) 313 [arXiv:hep-ph/0004028]; *Nucl. Phys. B* **B638** (2002) 165 [arXiv:hep-ph/0203131]; T. Nihei, L. Roszkowski and R. Ruiz de Austri, *JHEP* **0207** (2002) 024 [arXiv:hep-ph/0206266].
- [18] C. Boehm, A. Djouadi and M. Drees, *Phys. Rev. D* **62**, 035012 (2000) [arXiv:hep-ph/9911496]; J. R. Ellis, K. A. Olive and Y. Santoso, *Astropart. Phys.* **18**, 395 (2003) [arXiv:hep-ph/0112113]; J. Edsjo, M. Schelke, P. Ullio and P. Gondolo, *JCAP* **0304**, 001 (2003) [arXiv:hep-ph/0301106]; J. L. Diaz-Cruz, J. R. Ellis, K. A. Olive and Y. Santoso, *JHEP* **0705**, 003 (2007) [arXiv:hep-ph/0701229].
- [19] Joint LEP 2 Supersymmetry Working Group, *Combined LEP Chargino Results up to 208 GeV*,
http://lepsusy.web.cern.ch/lepsusy/www/inos_moriond01/charginos_pub.html.

- [20] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. **84**, 2322 (2000) [arXiv:hep-ph/9908309], and Phys. Rev. D **61**, 075005 (2000) [arXiv:hep-ph/9909334]; J. L. Feng, K. T. Matchev and F. Wilczek, Phys. Lett. B **482**, 388 (2000) [arXiv:hep-ph/0004043].
- [21] J. R. Ellis, K. A. Olive and P. Sandick, Phys. Lett. B **642**, 389 (2006) [arXiv:hep-ph/0607002], JHEP **0706**, 079 (2007) [arXiv:0704.3446 [hep-ph]], and JHEP **0808**, 013 (2008) [arXiv:0801.1651 [hep-ph]].
- [22] N. Polonsky and A. Pomarol, Phys. Rev. Lett. **73**, 2292 (1994) [arXiv:hep-ph/9406224], and Phys. Rev. D **51** (1995) 6532 [arXiv:hep-ph/9410231].
- [23] L. Calibbi, Y. Mambrini and S. K. Vempati, JHEP **0709**, 081 (2007) [arXiv:0704.3518 [hep-ph]]; L. Calibbi, A. Faccia, A. Masiero and S. K. Vempati, Phys. Rev. D **74**, 116002 (2006) [arXiv:hep-ph/0605139].
- [24] E. Carquin, J. Ellis, M. E. Gomez, S. Lola and J. Rodriguez-Quintero, JHEP **0905** (2009) 026 [arXiv:0812.4243 [hep-ph]].
- [25] P. Moxhay and K. Yamamoto, Nucl. Phys. B **256** (1985) 130; K. Grassie, Phys. Lett. B **159** (1985) 32; B. Gato, Nucl. Phys. B **278** (1986) 189. R. Barbieri and L. J. Hall, Phys. Lett. B **338** (1994) 212 [arXiv:hep-ph/9408406]; Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Rev. D **51** (1995) 1337 [arXiv:hep-ph/9406245]; H. Murayama, M. Olechowski and S. Pokorski, Phys. Lett. B **371** (1996) 57 [arXiv:hep-ph/9510327].
- [26] G. Senjanovic, arXiv:0912.5375 [hep-ph].
- [27] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, Phys. Rev. D **63**, 097302 (2001) [arXiv:hep-ph/0004166]; H. Baer, C. Balazs, J. K. Mizukoshi and X. Tata, Phys. Rev. D **63**, 055011 (2001) [arXiv:hep-ph/0010068]; H. Baer, C. Balazs, M. Brhlik, P. Mercadante, X. Tata and Y. Wang, Phys. Rev. D **64**, 015002 (2001) [arXiv:hep-ph/0102156]; G. A. Blair, W. Porod and P. M. Zerwas, Eur. Phys. J. C **27**, 263 (2003) [arXiv:hep-ph/0210058]; M. R. Buckley and H. Murayama, Phys. Rev. Lett. **97**, 231801 (2006) [arXiv:hep-ph/0606088]; J. A. Casas, A. Ibarra and F. Jimenez-Alburquerque, JHEP **0704**, 064 (2007) [arXiv:hep-ph/0612289].
- [28] V. Barger, D. Marfatia and A. Mustafayev, Phys. Lett. B **665**, 242 (2008) [arXiv:0804.3601 [hep-ph]]; M. E. Gomez, S. Lola, P. Naranjo and J. Rodriguez-Quintero, arXiv:0901.4013 [hep-ph]; K. Kadota, K. A. Olive and L. Velasco-Sevilla, Phys. Rev. D **79**, 055018 (2009) [arXiv:0902.2510 [hep-ph]]; K. Kadota and K. A. Olive, Phys. Rev. D **80**, 095015 (2009) [arXiv:0909.3075 [hep-ph]]; J. N. Esteves, S. Kaneko, J. C. Romao, M. Hirsch and W. Porod, Phys. Rev. D **80**, 095003 (2009) [arXiv:0907.5090 [hep-ph]]; V. Barger, D. Marfatia, A. Mustafayev and A. Soleimani, Phys. Rev. D **80**, 076004 (2009) [arXiv:0908.0941 [hep-ph]].
- [29] For a review, see for example P. Nath and P. Fileviez Perez, Phys. Rept. **441**, 191 (2007) [arXiv:hep-ph/0601023].

- [30] H. Baer, M. A. Diaz, P. Quintana and X. Tata, *JHEP* **0004**, 016 (2000) [arXiv:hep-ph/0002245].
- [31] ISAJET, by H. Baer, F. Paige, S. Protopopescu and X. Tata, arXiv:hep-ph/0312045.
- [32] J. R. Ellis and M. K. Gaillard, *Phys. Lett. B* **88**, 315 (1979); C. Panagiotakopoulos and Q. Shafi, *Phys. Rev. Lett.* **52**, 2336 (1984).
- [33] A. Dedes and K. Tamvakis, *Phys. Rev. D* **56**, 1496 (1997) [arXiv:hep-ph/9703374].
- [34] Information about this code is available from K. A. Olive: it contains important contributions from T. Falk, G. Ganis, A. Mustafayev, J. McDonald, K. A. Olive, P. Sandick, Y. Santoso and M. Srednicki.
- [35] M. Battaglia *et al.*, *Eur. Phys. J. C* **22**, 535 (2001) [arXiv:hep-ph/0106204]; M. Battaglia, A. De Roeck, J. R. Ellis, F. Gianotti, K. A. Olive and L. Pape, *Eur. Phys. J. C* **33**, 273 (2004) [arXiv:hep-ph/0306219]; A. De Roeck, J. R. Ellis, F. Gianotti, F. Moortgat, K. A. Olive and L. Pape, *Eur. Phys. J. C* **49**, 1041 (2007) [arXiv:hep-ph/0508198].
- [36] D. Matalliotakis and H. P. Nilles, *Nucl. Phys. B* **435** (1995) 115 [arXiv:hep-ph/9407251]; M. Olechowski and S. Pokorski, *Phys. Lett. B* **344**, 201 (1995) [arXiv:hep-ph/9407404]; V. Berezhinsky, A. Bottino, J. R. Ellis, N. Fornengo, G. Mignola and S. Scopel, *Astropart. Phys.* **5**, 1 (1996) [arXiv:hep-ph/9508249]; M. Drees, M. M. Nojiri, D. P. Roy and Y. Yamada, *Phys. Rev. D* **56**, 276 (1997) [Erratum-ibid. *D* **64** (1997) 039901] [arXiv:hep-ph/9701219]; M. Drees, Y. G. Kim, M. M. Nojiri, D. Toya, K. Hasuko and T. Kobayashi, *Phys. Rev. D* **63**, 035008 (2001) [arXiv:hep-ph/0007202]; P. Nath and R. Arnowitt, *Phys. Rev. D* **56**, 2820 (1997) [arXiv:hep-ph/9701301]; J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Schmitt, *Phys. Rev. D* **58**, 095002 (1998) [arXiv:hep-ph/9801445]; J. R. Ellis, T. Falk, G. Ganis and K. A. Olive, *Phys. Rev. D* **62** (2000) 075010 [arXiv:hep-ph/0004169]; A. Bottino, F. Donato, N. Fornengo and S. Scopel, *Phys. Rev. D* **63**, 125003 (2001) [arXiv:hep-ph/0010203]; D. Cerdeno and C. Munoz, *JHEP* **0410** (2004) 015, [arXiv:hep-ph/0405057].
- [37] J. Ellis, K. Olive and Y. Santoso, *Phys. Lett. B* **539**, 107 (2002) [arXiv:hep-ph/0204192]; J. R. Ellis, T. Falk, K. A. Olive and Y. Santoso, *Nucl. Phys. B* **652**, 259 (2003) [arXiv:hep-ph/0210205]; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *Phys. Rev. D* **71**, 095008 (2005) [arXiv:hep-ph/0412059]. H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *JHEP* **0507** (2005) 065, [arXiv:hep-ph/0504001]; J. R. Ellis, K. A. Olive and P. Sandick, *Phys. Rev. D* **78**, 075012 (2008) [arXiv:0805.2343 [hep-ph]].
- [38] S. P. Martin and M. T. Vaughn, *Phys. Rev. D* **50** (1994) 2282 [arXiv:hep-ph/9311340].
- [39] I. Gogoladze, R. Khalid, N. Okada and Q. Shafi, *Phys. Rev. D* **79**, 095022 (2009) [arXiv:0811.1187 [hep-ph]].

- [40] A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer and O. Vives, *Phys. Rev. D* **64**, 076009 (2001) [arXiv:hep-ph/0103324]; S. Profumo, *Phys. Rev. D* **68**, 015006 (2003) [arXiv:hep-ph/0304071]; B. Ananthanarayan and P. N. Pandita, *Int. J. Mod. Phys. A* **22**, 3229 (2007) [arXiv:0706.2560 [hep-ph]].
- [41] Tevatron Electroweak Working Group and the CDF and D0 Collaborations, arXiv:0903.2503 [hep-ex].
- [42] C. Amsler *et al.* [Particle Data Group], *Phys. Lett. B* **667**, 1 (2008).
- [43] C. Degrandi, P. Gambino and G. F. Giudice, *JHEP* **0012** (2000) 009 [arXiv:hep-ph/0009337], as implemented by P. Gambino and G. Ganis.
- [44] J. R. Ellis, S. Heinemeyer, K. A. Olive, A. M. Weber and G. Weiglein, *JHEP* **0708**, 083 (2007) [arXiv:0706.0652 [hep-ph]].
- [45] E. Barberio *et al.* [Heavy Flavor Averaging Group (HFAG)], arXiv:hep-ex/0603003.
- [46] [The Muon g-2 Collaboration], *Phys. Rev. Lett.* **92** (2004) 161802, [arXiv:hep-ex/0401008]; G. Bennett et al. [The Muon g-2 Collaboration], *Phys. Rev. D* **73** (2006) 072003 [arXiv:hep-ex/0602035].
- [47] M. Davier, S. Eidelman, A. Hocker and Z. Zhang, *Eur. Phys. J. C* **31** (2003) 503 [arXiv:hep-ph/0308213]; M. Knecht, *Lect. Notes Phys.* **629**, 37 (2004) [arXiv:hep-ph/0307239]; K. Melnikov and A. Vainshtein, *Phys. Rev. D* **70** (2004) 113006 [arXiv:hep-ph/0312226]; J. F. de Troconiz and F. J. Yndurain, *Phys. Rev. D* **71**, 073008 (2005) [arXiv:hep-ph/0402285]; M. Passera, arXiv:hep-ph/0411168; K. Hagiwara, A. Martin, D. Nomura and T. Teubner, *Phys. Lett. B* **649** (2007) 173 [arXiv:hep-ph/0611102]; M. Davier, *Nucl. Phys. Proc. Suppl.* **169**, 288 (2007) [arXiv:hep-ph/0701163]; J. Miller, E. de Rafael and B. Roberts, *Rept. Prog. Phys.* **70** (2007) 795 [arXiv:hep-ph/0703049]; S. Eidelman, talk given at the ICHEP06, Moscow, July 2006, see: http://ichep06.jinr.ru/reports/333_6s1.9p30_Eidelman.pdf; F. Jegerlehner and A. Nyffeler, *Phys. Rept.* **477**, 1 (2009) [arXiv:0902.3360 [hep-ph]]; T. Teubner, K. Hagiwara, R. Liao, A. D. Martin and D. Nomura, arXiv:1001.5401 [hep-ph].
- [48] M. Davier, A. Hoecker, B. Malaescu, C. Z. Yuan and Z. Zhang, arXiv:0908.4300 [hep-ph].
- [49] LEP Higgs Working Group for Higgs boson searches, OPAL Collaboration, ALEPH Collaboration, DELPHI Collaboration and L3 Collaboration, *Phys. Lett. B* **565** (2003) 61 [arXiv:hep-ex/0306033]. *Search for neutral Higgs bosons at LEP*, paper contributed to EPS-HEP 2005, Lisboa, Portugal, LHWG-NOTE-2005-01, ALEPH-2005-002, DELPHI-2005-021, L3-NOTE-2830, OPAL-TN-746, http://lephiggs.web.cern.ch/LEPHIGGS/papers/July2005_MSSM/index.html

- [50] S. Heinemeyer, W. Hollik and G. Weiglein, *Comput. Phys. Commun.* **124** (2000) 76 [arXiv:hep-ph/9812320]; S. Heinemeyer, W. Hollik and G. Weiglein, *Eur. Phys. J. C* **9** (1999) 343 [arXiv:hep-ph/9812472].
- [51] J. R. Ellis, J. Giedt, O. Lebedev, K. Olive and M. Srednicki, *Phys. Rev. D* **78**, 075006 (2008) [arXiv:0806.3648 [hep-ph]].
- [52] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, *Phys. Lett. B* **133**, 61 (1983); J. R. Ellis, C. Kounnas and D. V. Nanopoulos, *Nucl. Phys. B* **247**, 373 (1984).
- [53] J. R. Ellis, D. V. Nanopoulos and K. A. Olive, *Phys. Lett. B* **525**, 308 (2002) [arXiv:hep-ph/0109288].
- [54] J. R. Ellis, S. Heinemeyer, K. A. Olive and G. Weiglein, *JHEP* **0502**, 013 (2005) [arXiv:hep-ph/0411216].